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Grand unification in the spectral Pati-Salam model

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Abstract: We analyze the running at one-loop of the gauge couplings in the spectral Pati-Salam model that was derived in the framework of noncommutative geometry. There are a few different scenarios for the scalar particle content which are determined by the precise form of the Dirac operator for the finite noncommutative space. We consider these different scenarios and establish for all of them unification of the Pati-Salam gauge couplings. The boundary conditions are set by the usual RG flow for the Standard Model couplings at an intermediate mass scale at which the Pati-Salam symmetry is broken.

Keywords: Beyond Standard Model, Non-Commutative Geometry, Spontaneous Symmetry Breaking, Renormalization Group

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Contents

1 Introduction

This paper builds on two recent discoveries in the noncommutative geometry approach to particle physics: we showed in [12] how to obtain inner fluctuations of the metric without having to assume the order one condition on the Dirac operator. Moreover the original argument by classification [6] of finite geometries F that can provide the fine structure of Euclidean space-time as a product $M \times F$ (where M is a usual 4-dimensional Riemannian space) has now been replaced by a much stronger uniqueness statement [10, 11]. This new result shows that the algebra

$$M_2(\mathbb{H}) \oplus M_4(\mathbb{C}), \tag{1.1}$$

where $\mathbb{H}$ are the quaternions, appears uniquely when writing the higher analogue of the Heisenberg commutation relations. This analogue is written in terms of the basic ingredients of noncommutative geometry where one takes a spectral point of view, encoding geometry in terms of operators on a Hilbert space $\mathcal{H}$. In this way, the inverse line element is an unbounded self-adjoint operator D. The operator D is the tensor sum of the usual Dirac operator on M and a 'finite Dirac operator' on F, which is simply a hermitian matrix D_F. The usual Dirac operator involves γ matrices which allow one to combine the momenta into a single operator. The higher analogue of the Heisenberg relations puts the spatial variables on similar footing by combining them into a single operator Y using another set of γ matrices and it is in this process that the algebra (1.1) appears canonically and uniquely in dimension 4. We refer to [10, 11] for a detailed account. What matters for the present paper is that the above process leads without arbitrariness to the Pati-Salam [22] gauge group $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(4)$, together with the corresponding gauge fields and a scalar sector, all derived as inner perturbations of D [12]. Note that the scalar sector can not be chosen freely, in contrast to the early work on Pati-Salam unification [3, 13, 15, 16]. In fact, there are only a few possibilities for the precise scalar content, depending on the assumptions made on the finite Dirac operator.

From the spectral action principle, the dynamics and interactions are described by the *spectral action* [4, 5],

$$\mathrm{tr}(f(D_A/\Lambda)) \tag{1.2}$$

where Λ is a cutoff scale and f an even and positive function. In the present case, it can be expanded using heat kernel methods,

$$\mathrm{tr}(f(D_A/\Lambda)) \sim F_4\Lambda^4 a_0 + F_2\Lambda^2 a_2 + F_0 a_4 + \cdots \tag{1.3}$$

where F_4, F_2, F_0 are coefficients related to the function f and a_k are Seeley deWitt coefficients, expressed in terms of the curvature of M and (derivatives of) the gauge and scalar fields. This action is interpreted as an effective field theory for energies lower than Λ.

One important feature of the spectral action is that it gives the usual Pati-Salam action with unification of the gauge couplings [12] (cf. eq. (3.1) below). This is very similar to the case of the spectral Standard Model [9] where there is unification of gauge couplings. Since it is well known that the SM gauge couplings do not meet exactly, it is crucial to investigate the running of the Pati-Salam gauge couplings beyond the Standard Model and to find a scale Λ where there is grand unification:

$$g_R(\Lambda) = g_L(\Lambda) = g(\Lambda). \tag{1.4}$$

This would then be the scale at which the spectral action (1.3) is valid as an effective theory. There is a hierarchy of three energy scales: SM, an intermediate mass scale m_R where symmetry breaking occurs and which is related to the neutrino Majorana masses ($10^{11} - 10^{13}$GeV), and the GUT scale Λ.

For simplicity, we restrict our analysis to the running of the gauge couplings at one loop. Indeed, at two loops the gauge and scalar couplings are mixed and influence each other. Hence at the two loop level the running of the gauge couplings requires a complete understanding of the scalar sector and the scalar potential, which in turn requires a suitable method for dealing with quadratic divergences that appear from the mass terms. This deserves a careful study and goes beyond the aims of the present paper.

Thus, we analyze the running of the gauge couplings according to the usual (one-loop) RG equation where each takes the form

$$16\pi^2 \frac{dg}{dt} = -bg^3. \tag{1.5}$$

The coefficient b is determined by the particle content and their representation theory [14, 17–19] for which we use [20] as well as the program PyR@TE. As mentioned before, depending on the assumptions on D_F, one may vary to a limited extent the scalar particle content, consisting of either composite or fundamental scalar fields. We will not limit ourselves to a specific model but consider all cases separately. In fact, we establish grand unification for all of them, thus confirming validity of the spectral action at the corresponding scale, independent of the specific form of D_F.

Shortly after the submission of the present paper, the preprint [2] appeared in which our results are confirmed and confronted with recent data from LHC.

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2 Spectral Pati-Salam and grand unification

One of the pressing questions at present is whether there is new physics beyond the Standard Model. The success of the spectral construction of the Standard Model, predicting its particle content, including gauge fields, Higgs fields as well as a singlet whose vev gives Majorana mass to the right handed neutrino, is a strong signal that we are on the right track. The route that led to this conclusion starts with classifying the algebras of the finite space. The results show that the only algebras which solve the fermion doubling problem are of the form $M_{2a}(\mathbb{C}) \oplus M_{2a}(\mathbb{C})$ where a is an even integer. An arbitrary symplectic constraint is imposed on the first algebra restricting it from $M_{2a}(\mathbb{C})$ to $M_a(\mathbb{H})$. The first non-trivial algebra one can consider is for $a = 2$ with the algebra [6]

$$M_2(\mathbb{H}) \oplus M_4(\mathbb{C}). \tag{2.1}$$

Coincidentally, and as explained in the introduction, the above algebra comes out as a solution of the two-sided Heisenberg quantization relation between the Dirac operator D and the two maps from the four spin-manifold and the two four spheres $S^4 \times S^4$ [10, 11]. This removes the arbitrary symplectic constraint and replaces it with a relation that quantizes the four-volume in terms of two quanta of geometry and have far reaching consequences on the structure of space-time.

The existence of the chirality operator γ that commutes with the algebra breaks the quaternionic matrices $M_2(\mathbb{H})$ to the diagonal subalgebra and leads us to consider the finite algebra

$$\mathcal{A}_F = \mathbb{H}_R \oplus \mathbb{H}_L \oplus M_4(\mathbb{C}). \tag{2.2}$$

The Pati-Salam gauge group $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(4)$ is obtained as the inner automorphism group of $\mathcal{A} = \mathcal{C}^\infty(M) \otimes \mathcal{A}_F$, and the corresponding gauge bosons appear as inner perturbations of the (spacetime) Dirac operator [12].

Next, an element of the Hilbert space $\Psi \in \mathcal{H}$ is represented by

$$\Psi_M = \begin{pmatrix} \psi_A \\ \psi_{A'} \end{pmatrix}, \quad \psi_{A'} = \psi_A^c \tag{2.3}$$

where ψ_A^c is the conjugate spinor to ψ_A. Thus all primed indices A' correspond to the Hilbert space of conjugate spinors. It is acted on by both the left algebra $M_2(\mathbb{H})$ and the right algebra $M_4(\mathbb{C})$. Therefore the index A can take 16 values and is represented by

$$A = \alpha I \tag{2.4}$$

where the index α is acted on by quaternionic matrices and the index I by $M_4(\mathbb{C})$ matrices. Moreover, when the grading breaks $M_2(\mathbb{H})$ into $\mathbb{H}_R \oplus \mathbb{H}_L$ the index α is decomposed to $\alpha = \dot{a}, a$ where $\dot{a} = \dot{1}, \dot{2}$ (dotted index) is acted on by the first quaternionic algebra $\mathbb{H}_R$ and $a = 1, 2$ is acted on by the second quaternionic algebra $\mathbb{H}_L$. When $M_4(\mathbb{C})$ breaks into $\mathbb{C} \oplus M_3(\mathbb{C})$ (due to symmetry breaking or through the use of the order one condition as in [6]) the index I is decomposed into $I = 1, i$ and thus distinguishing leptons and quarks,

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where the 1 is acted on by the $\mathbb{C}$ and the i by $M_3(\mathbb{C})$. Therefore the various components of the spinor ψ_A are

$$\psi_{\alpha I} = \begin{pmatrix} \nu_R \ u_{iR} \ \nu_L \ u_{iL} \\ e_R \ d_{iR} \ e_L \ d_{iL} \end{pmatrix}, \qquad i = 1,2,3 \tag{2.5}$$
$$= (\psi_{\dot{a}1}, \psi_{\dot{a}i}, \psi_{a1}, \psi_{ai}), \qquad a = 1,2, \quad \dot{a} = \dot{1}, \dot{2}$$

This is a general prediction of the spectral construction that there is 16 fundamental Weyl fermions per family, 4 leptons and 12 quarks.

The (finite) Dirac operator can be written in matrix form

$$D_F = \begin{pmatrix} D_A^B & D_A^{B'} \\ D_{A'}^B & D_{A'}^{B'} \end{pmatrix}, \tag{2.6}$$

and must satisfy the properties

$$\gamma_F D_F = -D_F \gamma_F \qquad J_F D_F = D_F J_F \tag{2.7}$$

where $J_F^2 = 1$. Matrix realizations of γ_F and J_F are given by

$$\gamma_F = \begin{pmatrix} G_F & 0 \\ 0 & -\overline{G}_F \end{pmatrix}, \qquad G_F = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}, \qquad J_F = \begin{pmatrix} 0_4 & 1_4 \\ 1_4 & 0_4 \end{pmatrix} \circ \mathrm{cc} \tag{2.8}$$

where cc stands for complex conjugation. These relations, together with the hermiticity of D imply the relations

$$(D_F)_{A'}^{B'} = \left(\overline{D}_F\right)_A^B \qquad (D_F)_{A'}^B = \left(\overline{D}_F\right)_B^{A'} \tag{2.9}$$

and have the following zero components [7]

$$(D_F)_{aI}^{bJ} = 0 = (D_F)_{\dot{a}I}^{\dot{b}J} \tag{2.10}$$
$$(D_F)_{aI}^{\dot{b}'J'} = 0 = (D_F)_{\dot{a}I}^{b'J'} \tag{2.11}$$

leaving the components $(D_F)_{aI}^{\dot{b}J}$, $(D_F)_{aI}^{b'J'}$ and $(D_F)_{\dot{a}I}^{\dot{b}'J'}$ arbitrary. These restrictions lead to important constraints on the structure of the connection that appears in the inner fluctuations of the Dirac operator. In particular the operator D of the full noncommutative space given by

$$D = D_M \otimes 1 + \gamma_5 \otimes D_F \tag{2.12}$$

gets modified to

$$D_A = D + A_{(1)} + J A_{(1)} J^{-1} + A_{(2)} \tag{2.13}$$

where

$$A_{(1)} = \sum a\,[D,b]\,, \qquad A_2 = \sum \widehat{a}\left[A_{(1)}, \widehat{b}\right], \qquad \widehat{a} = J a J^{-1} \tag{2.14}$$

We have shown in [12] that components of the connection A which are tensored with the Clifford gamma matrices γ^{μ} are the gauge fields of the Pati-Salam model with the symmetry of $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(4)$. On the other hand, the non-vanishing components of the connection which are tensored with the gamma matrix γ_5 are given by

$$(A)_{aI}^{\dot{b}J} \equiv \gamma_5 \Sigma_{aI}^{\dot{b}J}, \qquad (A)_{aI}^{b'J'} = \gamma_5 H_{aIbJ}, \qquad (A)_{\dot{a}I}^{\dot{b}'J'} \equiv \gamma_5 H_{\dot{a}I\dot{b}J} \tag{2.15}$$

where $H_{aIbJ} = H_{bJaI}$ and $H_{\dot{a}I\dot{b}J} = H_{\dot{b}J\dot{a}I}$, which is the most general Higgs structure possible. These correspond to the representations with respect to $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{SU}(4)$:

$$\Sigma_{aI}^{\dot{b}J} = (2_R, 2_L, 1) + (2_R, 2_L, 15) \tag{2.16}$$

$$H_{aIbJ} = (1_R, 1_L, 6) + (1_R, 3_L, 10) \tag{2.17}$$

$$H_{\dot{a}I\dot{b}J} = (1_R, 1_L, 6) + (3_R, 1_L, 10) \tag{2.18}$$

We note, however, that the inner fluctuations form a semi-group and if a component $(D_F)_{aI}^{\dot{b}J}$ or $(D_F)_{aI}^{b'J'}$ or $(D_F)_{aI}^{\dot{b}'J'}$ vanish, then the corresponding A field will also vanish. We distinguish three cases: 1) Left-right symmetric Pati-Salam model with fundamental Higgs fields $\Sigma_{aI}^{\dot{b}J}$, H_{aIbJ} and $H_{\dot{a}I\dot{b}J}$. In this model the field H_{aIbJ} should have a zero vev. 2) A Pati-Salam model where the Higgs field H_{aIbJ} that couples to the left sector is set to zero which is desirable because there is no symmetry between the left and right sectors at low energies. 3) If one starts with $(D_F)_{aI}^{\dot{b}J}$ or $(D_F)_{aI}^{b'J'}$ or $(D_F)_{\dot{a}I}^{\dot{b}'J'}$ whose values are given by those that were derived for the Standard Model, then the Higgs fields $\Sigma_{aI}^{\dot{b}J}$, H_{aIbJ} and $H_{\dot{a}I\dot{b}J}$ will become composite and expressible in terms of more fundamental fields Σ_I^J, $\Delta_{\dot{a}J}$ and $\phi_{\dot{a}}^{b}$. We refer to this as the composite model.

Depending on the precise particle content we determine the coefficients b_R, b_L, b in (1.5) that control the RG flow of the Pati-Salam gauge couplings g_R, g_L, g. We run them to look for unification of the coupling $g_R = g_L = g$. The boundary conditions are taken at the intermediate mass scale $\mu = m_R$ to be the usual (e.g. [21, eq. (5.8.3)])

$$\frac{1}{g_1^2} = \frac{2}{3}\frac{1}{g^2} + \frac{1}{g_R^2}, \qquad \frac{1}{g_2^2} = \frac{1}{g_L^2}, \qquad \frac{1}{g_3^2} = \frac{1}{g^2}, \tag{2.19}$$

in terms of the Standard Model gauge couplings g_1, g_2, g_3. At the mass scale m_R the Pati-Salam symmetry is broken to that of the Standard Model, and we take it to be the same scale that is present in the see-saw mechanism. It should thus be of the order $10^{11} - 10^{13}$GeV. We now discuss the three models, in order of complexity.

2.1 Pati-Salam with composite Higgs fields

We first consider the case of a finite Dirac operator for which the Standard Model subalgebra $\mathbb{C} \oplus \mathbb{H}_L \oplus M_3(\mathbb{C}) \subset \mathcal{A}_F$ satisfies the first-order condition [6]. This condition is extremely constraining and forces the couplings of the right-handed neutrino to be with a singlet. In this case, the off-diagonal term in (2.6) becomes

$$D_{\alpha I}^{\beta' K'} = \delta_{\alpha}^{\dot{1}} \delta_{\dot{1}'}^{\beta'} \delta_I^1 \delta_{1'}^{K'} k^{*\nu_R}, \tag{2.20}$$

particle	$SU(2)_R$	$SU(2)_L$	$SU(4)$
$\phi^b_{\dot a}$	2	2	1
$\Delta_{\dot a I}$	2	1	4
Σ^I_J	1	1	15

Table 1. Pati-Salam scalar particle content and their representations for a first-order Dirac operator. The field Σ^I_J in the last row is decoupled if there is quark-lepton coupling unification.

and the diagonal structure of D_F is determined by the following sub-matrices [7]

$$D_{\alpha 1}{}^{\beta 1} = \begin{pmatrix} 0 & D^{\dot b 1}_{a 1} \\ D^{b 1}_{\dot a 1} & 0 \end{pmatrix}, \qquad D^{\dot b 1}_{a 1} = \left(D^{b 1}_{\dot a 1}\right)^* \equiv D^{\dot b}_{a(l)} \tag{2.21}$$
$$D_{\alpha i}{}^{\beta j} = \begin{pmatrix} 0 & D^{\dot b}_{a(q)}\delta^j_i \\ D^{b}_{\dot a(q)}\delta^j_i & 0 \end{pmatrix}, \qquad D^{b}_{\dot a(q)} = \left(D^{\dot b}_{a(q)}\right)^*$$

where

$$D^{\dot b}_{a(q)} = \begin{pmatrix} k^{*u} & 0 \\ 0 & k^{*d} \end{pmatrix}.$$

The Yukawa couplings k^ν, k^e, k^u, k^d are 3×3 matrices in generation space. Notice that this structure gives Dirac masses to all the fermions, but Majorana masses only for the right-handed neutrinos. One can also consider the special case of lepton and quark unification by equating $k^\nu = k^u, k^e = k^d$ which imply some simplifications.

The inner perturbations of the finite Dirac operator of the above type were determined in [12] to be composite fields $\Sigma^{bJ}_{\dot a I}$ and $H_{\dot a I \dot b J}$, depending on fundamental Higgs fields $\phi^b_{\dot a}$, Σ^J_I and $\Delta_{\dot a J}$ in the following way:

$$\Sigma^{bJ}_{\dot a I} = \left(k^\nu \phi^b_{\dot a} + k^e \widetilde{\phi}^b_{\dot a}\right)\Sigma^J_I + \left(k^u \phi^b_{\dot a} + k^d \widetilde{\phi}^b_{\dot a}\right)\left(\delta^J_I - \Sigma^J_I\right),$$
$$H_{\dot a I \dot b J} = k^{*\nu_R}\Delta_{\dot a J}\Delta_{\dot b I}. \tag{2.22}$$

The field $\widetilde{\phi}^b_{\dot a}$ is not an independent field and is given by

$$\widetilde{\phi}^b_{\dot a} = \sigma_2 \overline{\phi}^b_{\dot a}\sigma_2. \tag{2.23}$$

We have listed the fundamental Higgs fields and their representations in table 1. We first assume that there is lepton quark unification, so that the Σ^J_I is decoupled.

Before turning to the computation of the β-functions of the Pati-Salam gauge couplings for the composite model, let us discuss the scalar sector that remains after spontaneous symmetry breaking to the Standard Model gauge group. A quick analysis leads to the scalar fields listed in table 2. Note that this includes the SM Higgs and a real scalar singlet. The latter played a key role in [7] in lowering the Higgs mass prediction to a realistic value [8].

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	$\mathrm{U}(1)_Y$	$\mathrm{SU}(2)_L$	$\mathrm{SU}(3)$
$\begin{pmatrix}\phi_1^0\\ \phi_1^+\end{pmatrix} = \begin{pmatrix}\phi_1^1\\ \phi_1^2\end{pmatrix}$	1	2	1
$\begin{pmatrix}\phi_2^-\\ \phi_2^0\end{pmatrix} = \begin{pmatrix}\phi_2^1\\ \phi_2^2\end{pmatrix}$	-1	2	1
σ	0	1	1
η	$-\frac{2}{3}$	1	3

Table 2. Scalar particle content induced by the composite model with SM-representations.

A qualitative study of the form of the scalar potential that we have done for the present Pati-Salam composite model indicates that this result continues to hold here. However, being interested mainly in the running of the gauge couplings, we leave a full study of the potential and its physical implications for future work.

The presence of the above scalar fields of course also have an influence on the running of the Standard Model gauge couplings (at one loop). We compute that instead of the usual β-functions $(b_1, b_2, b_3) = \left(-\frac{41}{6}, \frac{19}{6}, 7\right)$ we have

$$(b_1, b_2, b_3) = \left(-\frac{64}{9}, 3, \frac{41}{6}\right).$$

One observes that this difference is relatively small (less than 5%). In fact, the scalar fields that appear in addition to the SM Higgs have a negligible effect in our study of the running of the gauge couplings below. Moreover, if some of the scalar fields have a mass of order m_R or higher, they are decoupled [1]. Note that this is in contrast to the effect on the running of the scalar couplings where, as already mentioned, the additional fields significantly change the physics of the Higgs sector. As said, the full analysis lies beyond the scope of this paper, but we refer to [8] for the relevant example.

Next, we compute the β-functions for the Pati-Salam couplings g_R, g_L, g in the presence of the above composite particle content (cf. table 1):

$$(b_R, b_L, b) = \left(\frac{7}{3}, 3, \frac{31}{3}\right). \tag{2.24}$$

The solutions of the RG-equations are easily found to be

$$g_R(\mu)^{-2} = g_R(m_R)^{-2} + \frac{1}{8\pi^2}\frac{7}{3}\log\frac{\mu}{m_R}, \tag{2.25}$$

$$g_L(\mu)^{-2} = g_L(m_R)^{-2} + \frac{1}{8\pi^2}3\log\frac{\mu}{m_R}, \tag{2.26}$$

$$g(\mu)^{-2} = g(m_R)^{-2} + \frac{1}{8\pi^2}\frac{31}{3}\log\frac{\mu}{m_R}, \tag{2.27}$$

We impose the boundary conditions (2.19) at the mass scale $\mu = m_R$.

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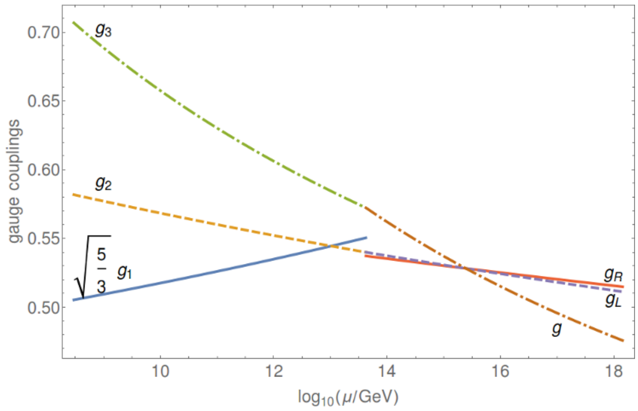

Figure 1. Running of coupling constants for the spectral Pati-Salam model with composite Higgs fields: g_1, g_2, g_3 for $\mu < m_R$ and g_R, g_L, g for $\mu > m_R$ with unification scale $\Lambda \approx 2.5 \times 10^{15}$ GeV for $m_R = 4.25 \times 10^{13}$ GeV.

Note that in our analysis we have disregarded the non-renormalizable, order eight terms that appear in the expansion of the spectral action for the composite model [12, section 8], so let us argue why they can be ignored. In fact, since we consider only the running of the gauge couplings at the one loop level, we can safely ignore these non-renormalizable terms. Moreover, their contribution to the running of other (scalar) couplings will be suppressed by negative powers of m_R, at least at the one loop level.

Our approach for finding a unification scale is as follows. We search for an energy scale where the couplings g_R, g_L and g are equal by varying the scale m_R at which the boundary conditions (2.19) are imposed. With the running of the Pati-Salam couplings governed by the coefficients (2.24) there is a unique value of m_R for which the three lines meet. The unification scale is $\Lambda \approx 2.5 \times 10^{15}$GeV and the value found for the intermediate scale is $m_R = 4.25 \times 10^{13}$GeV (figure 1).

If the scalar field Σ_I^J is not decoupled — in other words, if there is no lepton-quark coupling unification — then there is an additional scalar $(1_R, 1_L, 15)$ irreducible representation contributing to the β-function, giving a slightly different $(b_R, b_L, b) = \left(\frac{7}{3}, 3, 9\right)$. This in turn gives a unification scale $\Lambda \approx 6.3 \times 10^{15}$GeV for $m_R = 4.1 \times 10^{13}$GeV.

Let us conclude this subsection by mentioning that because of the assumptions made in our analysis, we trust these values only as indicative of the corresponding orders of magnitudes.

2.2 Pati-Salam with fundamental Higgs fields

Next, we consider the case of a more general finite Dirac operator, not satisfying the first-order condition with respect to the Standard Model subalgebra. We begin with the special case where

$$(D_F)_{aI}^{b'J'} = 0 \tag{2.28}$$

which implies that the Higgs field $H_{aIbJ} = 0$. The inner perturbations Σ_{aI}^{bJ} and $H_{\dot{a}I\dot{b}J}$ are now themselves fundamental Higgs fields [12, section 5] and their representations are listed in table 3.

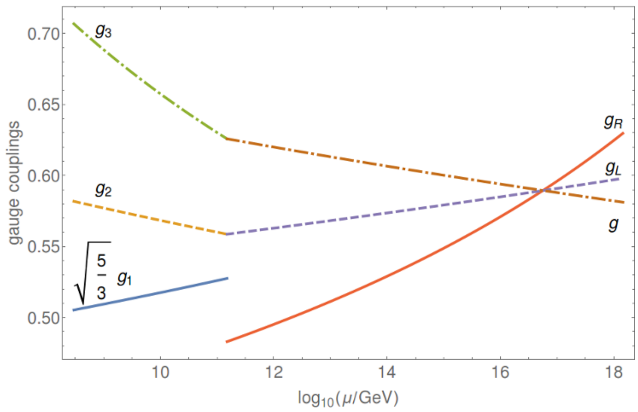

Figure 2. Running of coupling constants for the spectral Pati-Salam model with fundamental Higgs fields: g_1, g_2, g_3 for $\mu < m_R$ and g_R, g_L, g for $\mu > m_R$ with unification scale $\Lambda \approx 6.3 \times 10^{16}$ GeV for $m_R = 1.5 \times 10^{11}$ GeV.

particle	$SU(2)_R$	$SU(2)_L$	$SU(4)$
$\Sigma^{bJ}_{\dot{a}J}$	2	2	1 + 15
$H_{\dot{a}I\dot{b}J}$	3	1	10
	1	1	6

Table 3. Pati-Salam scalar particle content and their representations for a general finite Dirac operator.

The contribution of the enlarged scalar sector on the running of the SM gauge couplings can be ignored if we assume that most of them are heavy (mass of the order m_R) so that we can apply the decoupling theorem of [1]. The remaining low-dimensional representations that describe the 'light' scalars have a small contribution to the running of the SM gauge couplings and can be safely ignored, also since we are mainly interested in the order of magnitudes for the intermediate scale and the unification scale.

With the scalar content of table 3 we determine the Pati-Salam β-functions to be

$$(b_R, b_L, b) = \left(-\frac{26}{3}, -2, 2\right) \tag{2.29}$$

Note that the $SU(2)_R$ and $SU(2)_L$-sectors are not asymptotically free, due to the large scalar sector. Nevertheless, we can still run the gauge couplings with the boundary values set by (2.19).

Adopting the same approach as in the previous section for finding a unification scale, we arrive at figure 2. The unification scale is $\Lambda \approx 6.3 \times 10^{16}$GeV if we set $m_R = 1.5 \times 10^{11}$GeV.

2.3 Left-right symmetric Pati-Salam with fundamental Higgs fields

As a final possibility we consider the most general case for D_F which gives in addition to the fundamental Higgs fields in table 3 the field H_{aIbJ} in the $(1_R, 3_L, 10) + (1_R, 1_L, 6)$

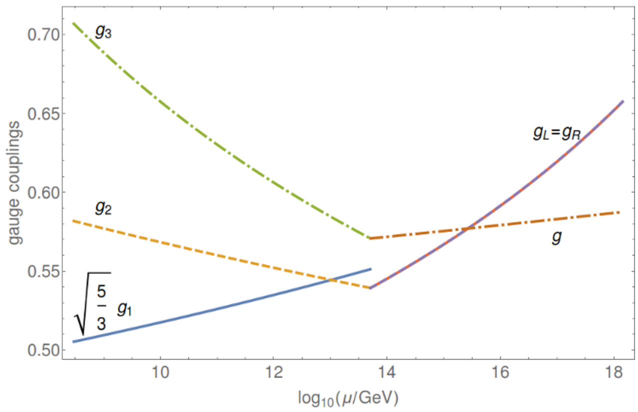

Figure 3. Running of coupling constants for the left-right symmetric spectral Pati-Salam model: g_1, g_2, g_3 for $\mu < m_R$ and g_R, g_L, g for $\mu > m_R$ with unification scale $\Lambda \approx 2.7 \times 10^{15}$ GeV for $m_R = 5.1 \times 10^{13}$ GeV.

representation. The β-functions become

$$(b_R, b_L, b) = \left(-\frac{26}{3}, -\frac{26}{3}, -\frac{4}{3}\right) \tag{2.30}$$

Since we are again only interested in the order of magnitude for m_R and Λ, we adopt the same approximations on the structure of the scalar sector as in the previous section. The scale m_R is determined so as to have left-right symmetry $g_R(m_R) = g_L(m_R)$. This happens for $m_R = 5.1 \times 10^{13}$GeV, resulting in figure 3. We find the unification scale to be $\Lambda \approx 2.7 \times 10^{15}$GeV.

3 Conclusions

We have analyzed the running of the Pati-Salam gauge couplings for the spectral model, considering different scalar field contents corresponding to the assumptions made on the finite Dirac operator. We stress that the number of possible models is quite restrictive and that one can not freely choose the particle content. We have identified the three main models, although there exists small variations on them. The different possibilities correspond to restrictions on the geometry of the finite space F. In all the models considered here, we establish unification of the gauge couplings, with boundary conditions set by the usual Standard Model gauge couplings at an intermediate mass scale.

Besides the direct physical interest of such grand unification, it also determines the scale at which the asymptotic expansion of Equation (1.3) is actually valid as an effective theory. In order to see this, note that the scale-invariant term $F_0 a_4$ in (1.3) for the spectral Pati-Salam model contains the terms [12]:

$$\frac{F_0}{2\pi^2} \int \left(g_L^2 \left(W_{\mu\nu L}^{\alpha}\right)^2 + g_R^2 \left(W_{\mu\nu R}^{\alpha}\right)^2 + g^2 \left(V_{\mu\nu}^{m}\right)^2 \right). \tag{3.1}$$

Normalizing this to give the Yang-Mills Lagrangian demands

$$\frac{F_0}{2\pi^2} g_L^2 = \frac{F_0}{2\pi^2} g_R^2 = \frac{F_0}{2\pi^2} g^2 = \frac{1}{4}, \tag{3.2}$$

which requires gauge coupling unification, $g_R = g_L = g$. Note that the similar result for the Standard Model gauge couplings does not hold (at least at the one-loop level) because the three couplings actually do not meet, even though they are required to be unified in the spectral action [4]. We consider this to be strong evidence for the spectral Pati-Salam model as a realistic possibility to go beyond the Standard Model.

To summarize, the spectral construction of particle physics models based on a spectral triple with a noncommutative space with metric dimension four and whose finite part has KO dimension 6 leads directly to a family of Pati-Salam models with gauge symmetry $SU(2)_R \times SU(2)_L \times SU(4)$ and well-defined Higgs structure. Breaking of $SU(2)_R \times SU(4)$ to $U(1) \times SU(3)$ occurs at some scale $m_R \sim 10^{11} - 10^{13}$ GeV with a unification scale where the three coupling constants meet of the order of 10^{16} GeV. All these breakings will have the Standard Model as an effective theory at low energies.

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