Grand Unification in the Spectral Pati-Salam Model

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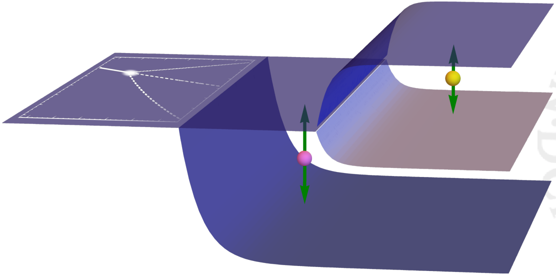

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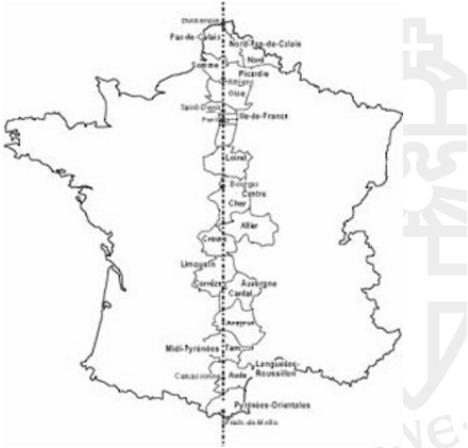
Contents

Meter defined in 1791 as 10^{-7} times one quarter of the meridian of the Earth.

Expedition in 1792: measuring the arc of the meridian between Barcelona-Duinkerken, at the beginning of the French revolution... [a]

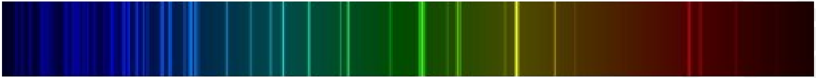


[a] Adler (2002)

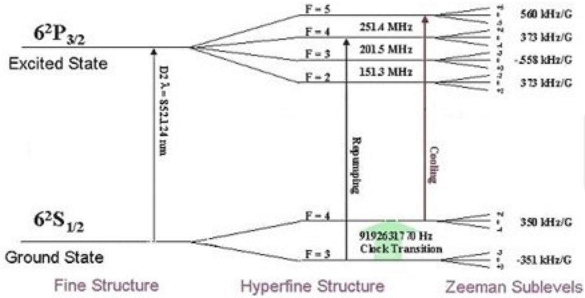
Meter made concrete by platinum bar “mètre-étalon”, saved (from 1889) in Pavillon de Breteuil near Paris:

Practical objections mètre-étalon (natural variations):

- 1960: meter defined as a multiple of a transition wavelength in Krypton 86Kr:

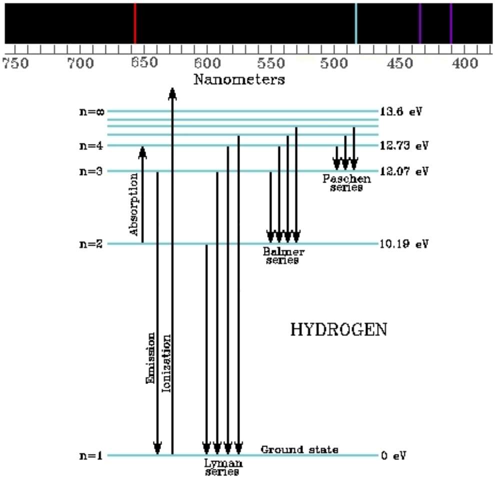


- 1967: **second** = 9192631770 periods of a transition radiation between two hyperfine levels in Caesium-133.



- 1983: Definition of the **meter** as the distance that light travels in 1/299792458 second...

So, measuring distances by looking at spectra



A fermion in a spacetime background

Minimal ingredients to describe a free fermion:

- coordinates on spacetime M:

$$x_\mu \cdot x_\nu(p) = x_\mu(p)x_\nu(p), etc.,$$

  with $\mu, \nu = 1, \ldots, 4$.
- propagation, described by Dirac operator $\partial\!\!\!/_M = i\gamma^\mu \partial_\mu$, acting on wavefunctions ψ:

$$S[\psi] = \int \overline{\psi} \partial\!\!\!/_M \psi \qquad \rightsquigarrow \text{EOM: } \partial\!\!\!/_M \psi = 0.$$

- This combination of coordinate algebra and operators is central to the spectral, or noncommutative approach [C 1994].

Emerging bosons

Our fermionic starting point induces a bosonic theory:

- "Inner fluctuations" by the coordinates [C 1996]:

$$\partial\!\!\!/_M \rightsquigarrow \partial\!\!\!/_M + \sum_j a_j[\partial\!\!\!/_M, a'_j]$$

  for functions a_j, a'_j depending on the coordinates x_μ.

- Then, by the chain rule:

$$\sum_j a_j[\partial\!\!\!/_M, a'_j] = A^\nu \gamma^\mu (\partial_\mu x^\nu) = A^\mu \gamma_\mu$$

  where A^μ is the electromagnetic 4-potential describing the photon.

Moreover, it is possible to derive a bosonic action from the (Euclidean) Dirac operator via the spectral action [CC 1996]:

$$\mathrm{Trace}\, e^{-\partial\!\!\!/_M^2/\Lambda^2} \sim c_4 \Lambda^4 \mathrm{Vol}(M) + c_2 \Lambda^2 \int R\sqrt{g} + c_0 \int (\partial_{[\mu} A_{\nu]})^2 + \cdots$$

for some coefficients $c_4, c_2, \ldots$.

We recognize

- The Einstein-Hilbert action $\int R\sqrt{g}$ for (Euclidean) gravity
- The Lagrangian $\int (\partial_{[\mu} A_{\nu]})^2$ for the electromagnetic field

Noncommutative fine structure of spacetime

Replace spacetime by
spacetime $\times$ **noncommutative space**: $M \times F$

- F is considered as internal space (Kaluza–Klein like)
- F is described by noncommutative matrices, that play the role of coordinates, just as spacetime is described by $x_\mu(p)$.
- 'Propagation' of particles in F is described by a 'Dirac operator' $\not{\partial}_F$ which is actually simply a hermitian matrix.

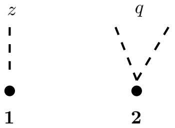
Note that the spectral approach is now the only way to describe the geometry of F.

Coordinates on F are elements in $\mathbb{C} \oplus \mathbb{H}$

- A complex number z
- A quaternion $q = q_0 + i q_k \sigma^k$; in terms of Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

It describes a two-point space, with internal structure:



Gauge group is given by unitaries: $U(1) \times SU(2)$.

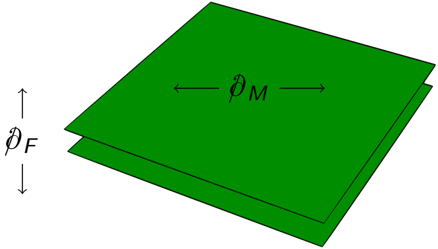
'Dirac operator'

$$\partial\!\!\!/_F = \begin{pmatrix} 0 & \overline{c} & 0 \\ c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- "Inner fluctuations" can be defined as before but now yield:

$$\sum_j \begin{pmatrix} z_j & 0 \\ 0 & q_j \end{pmatrix} \left[\partial\!\!\!/_F, \begin{pmatrix} z_j' & 0 \\ 0 & q_j' \end{pmatrix} \right] =: \begin{pmatrix} 0 & \overline{c}\overline{\phi_1} & \overline{c}\overline{\phi_2} \\ c\phi_1 & 0 & 0 \\ c\phi_2 & 0 & 0 \end{pmatrix}$$

Almost-commutative spacetimes



We combine this mild (matrix) noncommutativity with spacetime:

- coordinates of the almost-commutative spacetime $M \times F$:

$$\boxed{\hat{x}^\mu(p) = (z^\mu(p), q^\mu(p))}$$

  as elements in $\mathbb{C} \oplus \mathbb{H}$ (for each μ and each point p of M)
- The combined Dirac operator becomes

$$\boxed{D\!\!\!/_{M\times F} = D\!\!\!/_M + \gamma_5 D\!\!\!/_F}$$

Note that $D\!\!\!/^2_{M\times F} = D\!\!\!/^2_M + D\!\!\!/^2_F$, which will be useful later on.

Inner fluctuations on $M \times F$

So, we describe $M \times F$ by:

$$\hat{x}^\mu = (z^\mu, q^\mu)\,; \qquad \not{\partial}_{M\times F} = \not{\partial}_M + \gamma_5 \not{\partial}_F$$

As before, we consider inner fluctuations of $\not{\partial}_{M\times F}$ by $\hat{x}^\mu(p)$:

- The inner fluctuations of $\not{\partial}_F$ become scalar fields ϕ_1, ϕ_2.
- The inner fluctuations of $\not{\partial}_M$ become matrix-valued:

$$\sum_j a_j[\not{\partial}_M, a_j'] = a_\nu \gamma^\mu (\partial_\mu \hat{x}^\nu) =: \not{\partial}_M + A_\mu \gamma^\mu$$

with A_μ taking values in $\mathbb{C} \oplus \mathbb{H}$:

$$A_\mu = \begin{pmatrix} B_\mu & 0 & 0 \\ 0 & W_\mu^3 & W_\mu^+ \\ 0 & W_\mu^- & -W_\mu^3 \end{pmatrix}$$

corresponding to hypercharge and the W-bosons.

Action functional: electroweak theory

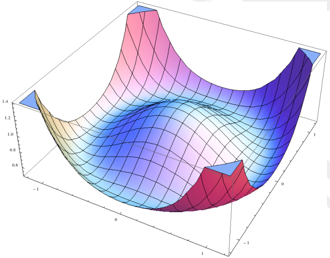
Use $\slashed{D}^2_{M\times F} = \slashed{D}^2_M + \slashed{D}^2_F$ to compute the spectral action

$$\mathrm{Trace}\, e^{-\slashed{D}^2_{M\times F}/\Lambda^2} = \mathrm{Trace}\, e^{-\slashed{D}^2_M/\Lambda^2}\left(1 - \frac{\slashed{D}^2_F}{\Lambda^2} + \frac{1}{2}\frac{\slashed{D}^4_F}{\Lambda^4} - \cdots\right)$$

$$\sim \left(c_4\Lambda^4 \mathrm{Vol}(M) + c_2\Lambda^2 \int R\sqrt{g} + c_0 \int F_{\mu\nu}F^{\mu\nu}\right)\left(1 - \frac{|\phi|^2}{\Lambda^2} + \frac{|\phi|^4}{2\Lambda^4}\right) + \cdots$$

We now recognize in terms of the field-strength $F_{\mu\nu}$ for A_μ:

- The Yang–Mills term $F_{\mu\nu}F^{\mu\nu}$ for hypercharge and W-boson
- The Higgs potential $-c_4\Lambda^2|\phi|^2 + \frac{1}{2}c_4|\phi|^4$



Describe $M \times F_{SM}$ by [CCM 2007]

- Coordinates: $\hat{x}^\mu(p) \in \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ (with unimodular unitaries $U(1)_Y \times SU(2)_L \times SU(3)$).
- Dirac operator $\not{\partial}_{M\times F} = \not{\partial}_M + \gamma_5 \not{\partial}_F$ where

$$\not{\partial}_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$$

is a 96×96-dimensional hermitian matrix where 96 is:

$$\underset{\text{families}}{3} \times \underset{\text{anti-particles}}{2} \times \big(\underset{(\nu_L, e_L)}{\underline{2} \otimes \underline{1}} + \underset{\nu_R}{\underline{1} \otimes \underline{1}} + \underset{e_R}{\underline{1} \otimes \underline{1}} + \underset{(u_L, d_L)}{\underline{2} \otimes \underline{3}} + \underset{u_R}{\underline{1} \otimes \underline{3}} + \underset{d_R}{\underline{1} \otimes \underline{3}} \big)$$

The Dirac operator on F_{SM}

$$\partial\!\!\!/_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$$

- The operator S is given by

$$S_l := \begin{pmatrix} 0 & 0 & Y_\nu & 0 \\ 0 & 0 & 0 & Y_e \\ Y_\nu^* & 0 & 0 & 0 \\ 0 & Y_e^* & 0 & 0 \end{pmatrix}, \quad S_q \otimes 1_3 = \begin{pmatrix} 0 & 0 & Y_u & 0 \\ 0 & 0 & 0 & Y_d \\ Y_u^* & 0 & 0 & 0 \\ 0 & Y_d^* & 0 & 0 \end{pmatrix} \otimes 1_3,$$

  where Y_ν, Y_e, Y_u and Y_d are 3×3 mass matrices acting on the three generations.

- The symmetric operator T only acts on the right-handed (anti)neutrinos, $T\nu_R = Y_R\overline{\nu_R}$ for a 3×3 symmetric Majorana mass matrix Y_R, and $Tf = 0$ for all other fermions $f \neq \nu_R$.

Inner fluctuations

Just as before, we find

- Inner fluctuations of $\partial\!\!\!/_M$ give a matrix

$$A_\mu = \begin{pmatrix} B_\mu & 0 & 0 & 0 \\ 0 & W_\mu^3 & W_\mu^+ & 0 \\ 0 & W_\mu^- & -W_\mu^3 & 0 \\ 0 & 0 & 0 & (G_\mu^a) \end{pmatrix}$$

  corresponding to hypercharge, weak and strong interaction.

- Inner fluctuations of $\partial\!\!\!/_F$ give

$$\begin{pmatrix} Y_\nu & 0 \\ 0 & Y_e \end{pmatrix} \rightsquigarrow \begin{pmatrix} Y_\nu \phi_1 & -Y_e \overline{\phi}_2 \\ Y_\nu \phi_2 & Y_e \overline{\phi}_1 \end{pmatrix}$$

  corresponding to SM-Higgs field. Similarly for Y_u, Y_d.

Dynamics and interactions

If we reconsider the spectral action:

$$\mathrm{Trace}\, e^{-\not{\partial}^2_{M\times F}/\Lambda^2} \sim \left(c_4\Lambda^4 \mathrm{Vol}(M) + c_0 \int F_{\mu\nu}F^{\mu\nu}\right)\left(1 - \frac{|\phi|^2}{\Lambda^2} + \frac{|\phi|^4}{2\Lambda^4}\right) + \cdot$$

we observe [CCM 2007]:

- The coupling constants of hypercharge, weak and strong interaction are expressed in terms of the single constant c_0 which implies

  $$\boxed{g_3^2 = g_2^2 = \frac{5}{3} g_1^2}$$

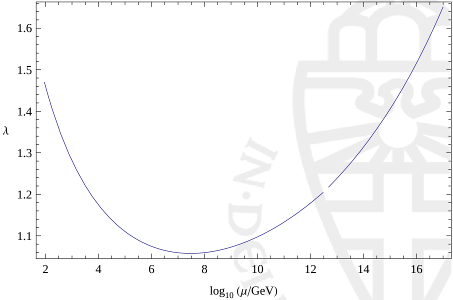
  In other words, there should be grand unification.
- Moreover, the quartic Higgs coupling λ is related via

$$\lambda \approx 24 \frac{3+\rho^4}{(3+\rho^2)^2} g_2^2; \qquad \rho = \frac{m_\nu}{m_{\mathrm{top}}}$$

This can be used to derive predictions as follows:

- Interpret the spectral action as an effective field theory at $\Lambda_{\text{GUT}} \approx 10^{13} - 10^{16}$ GeV.
- Run the quartic coupling constant λ to SM-energies to predict

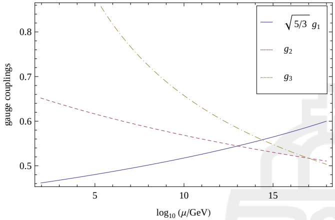
$$m_h^2 = \frac{4\lambda M_W^2}{3g_2^2}$$

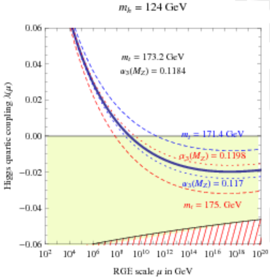


This gives [CCM 2007]

$$\boxed{167 \text{ GeV} \leq m_h \leq 176 \text{ GeV}}$$

1. This prediction is falsified by the measured value.
2. In the Standard Model there is not the presumed grand unification.
3. There is a problem with the low value of m_h, making the Higgs vacuum un/metastable [Elias-Miro et al. 2011].





- The matrix coordinates of the Standard Model arise naturally as a restriction of the following coordinates

$$\hat{x}^\mu(p) = \left(q_R^\mu(p), q_L^\mu(p), m^\mu(p)\right) \in \mathbb{H}_R \oplus \mathbb{H}_L \oplus M_4(\mathbb{C})$$

corresponding to a Pati–Salam unification:

$$U(1)_Y \times SU(2)_L \times SU(3) \to SU(2)_R \times SU(2)_L \times SU(4)$$

- The 96 fermionic degrees of freedom are structured as

$$\left(\begin{array}{cc|cc} \nu_R & u_{iR} & \nu_L & u_{iL} \\ e_R & d_{iR} & e_L & d_{iL} \end{array}\right) \qquad (i = 1, 2, 3)$$

- Again the finite Dirac operator is a 96×96-dimensional matrix (details in [CCS 2013]).

Inner fluctuations

- Inner fluctuations of $\partial\!\!\!/_M$ now give three gauge bosons:

$$W_R^\mu, \qquad W_L^\mu, \qquad V^\mu$$

  corresponding to $SU(2)_R \times SU(2)_L \times SU(4)$.
- For the inner fluctuations of $\partial\!\!\!/_F$ we distinguish two cases, depending on the initial form of $\partial\!\!\!/_F$:
  - I The Standard Model $\partial\!\!\!/_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$
  - II A more general $\partial\!\!\!/_F$ with zero $\overline{f}_L - f_L$-interactions.

Case I For a SM $\not{\partial}_F$, the resulting scalar fields are composite fields, expressed in scalar fields whose representations are:

	$SU(2)_R$	$SU(2)_L$	$SU(4)$
$\phi_{\dot{a}}^{b}$	2	2	1
$\Delta_{\dot{a}I}$	2	1	4
Σ_J^I	1	1	15

Case II For a more general finite Dirac operator, we have fundamental scalar fields:

particle	$SU(2)_R$	$SU(2)_L$	$SU(4)$
$\Sigma_{\dot{a}J}^{bJ}$	2	2	$1+15$
$H_{\dot{a}I\dot{b}J}$	3	1	10
	1	1	6

Action functional

As for the Standard Model, we can compute the spectral action which describes the usual Pati–Salam model with

- unification of the gauge couplings

$$g_R = g_L = g.$$

- A rather involved, fixed scalar potential, still subject to further study

However, independently from the spectral action, we can analyze the running at one loop of the gauge couplings [CCS 2015]:

1. We run the Standard Model gauge couplings up to a presumed PS $\to$ SM symmetry breaking scale m_R
2. We take their values as boundary conditions to the Pati–Salam gauge couplings g_R, g_L, g at this scale via

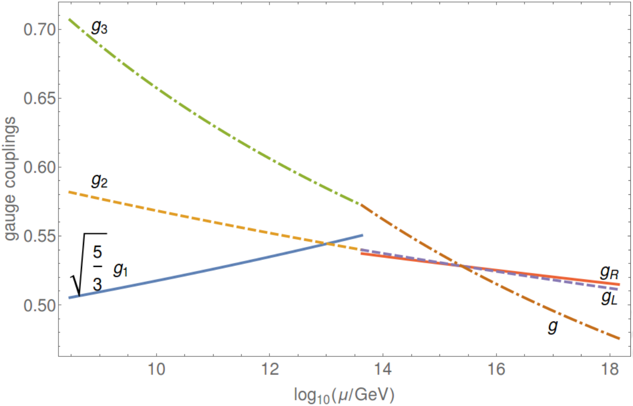
$$\frac{1}{g_1^2} = \frac{2}{3}\frac{1}{g^2} + \frac{1}{g_R^2}, \qquad \frac{1}{g_2^2} = \frac{1}{g_L^2}, \qquad \frac{1}{g_3^2} = \frac{1}{g^2},$$

3. Vary m_R in a search for a unification scale Λ where

$$g_R = g_L = g$$

   which is where the spectral action is valid as an effective theory.

For the Standard Model Dirac operator, we have found that with $m_R \approx 4.25 \times 10^{13}$ GeV there is unification at $\Lambda \approx 2.5 \times 10^{15}$ GeV:

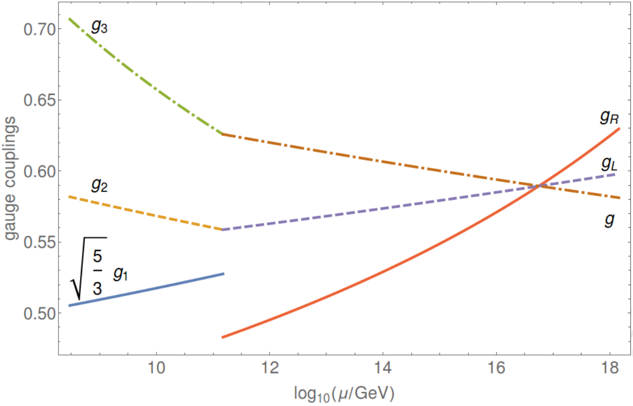


In this case, we can also say something about the scalar particles that remain after SSB:

	$U(1)_Y$	$SU(2)_L$	$SU(3)$
$\begin{pmatrix} \phi_1^0 \\ \phi_1^+ \end{pmatrix} = \begin{pmatrix} \phi_{\dot{1}}^{\dot{1}} \\ \phi_{\dot{1}}^{\dot{2}} \end{pmatrix}$	1	2	1
$\begin{pmatrix} \phi_2^- \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \phi_{\dot{2}}^{\dot{1}} \\ \phi_{\dot{2}}^{\dot{2}} \end{pmatrix}$	-1	2	1
σ	0	1	1
η	$-\frac{2}{3}$	1	3

- It turns out that these scalar fields have a little influence on the running of the SM-gauge couplings (at one loop).
- However, this sector contains the real scalar singlet σ that allowed for a realistic Higgs mass and that stabilizes the Higgs vacuum [CC 2012].

For the more general case, we have found that with $m_R \approx 1.5 \times 10^{11}$ GeV there is unification at $\Lambda \approx 6.3 \times 10^{16}$ GeV:



Conclusion

With our walk through the noncommutative garden, we have arrived at a spectral Pati–Salam model that

- goes beyond the Standard Model
- has a fixed scalar sector once the finite Dirac operator has been fixed (only a few scenarios)
- exhibits grand unification for all of these scenarios (confirmed by [Aydemir–Minic–Sun–Takeuchi 2015])
- the scalar sector has the potential to stabilize the Higgs vacuum and allow for a realistic Higgs mass.

Further reading

A. Chamseddine, A. Connes, WvS.

Beyond the Spectral Standard Model: Emergence of Pati-Salam Unification. *JHEP* 11 (2013) 132. [arXiv:1304.8050]

Grand Unification in the Spectral Pati-Salam Model. To appear in *JHEP* [arXiv:1507.08161]

WvS.

Noncommutative Geometry and Particle Physics. Mathematical Physics Studies, Springer, 2015.

and also: `http://www.noncommutativegeometry.nl`