Beyond the spectral Standard Model: Pati-Salam unification

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Overview

- Motivation: NCG and HEP
- Noncommutative Riemannian spin manifolds (aka spectral triples)
- Gauge theory from spectral triples: gauge group, gauge fields
- The spectral Standard Model and Beyond
A fermion in a spacetime background

- Spacetime is a (pseudo) Riemannian manifold $M$: local coordinates $x_\mu$ generate algebra $C^\infty(M)$.

- Propagator is described by Dirac operator $D_M$, essentially a 'square root' of the Laplacian.
The circle

- The **Laplacian** on the circle $S^1$ is given by

$$\Delta_{S^1} = -\frac{d^2}{dt^2}; \quad (t \in [0, 2\pi])$$

- The **Dirac operator** on the circle is

$$D_{S^1} = -i\frac{d}{dt}$$

with square $\Delta_{S^1}$. 
The 2-dimensional torus

- Consider the two-dimensional torus $\mathbb{T}^2$ parametrized by two angles $t_1, t_2 \in [0, 2\pi)$.
- The Laplacian reads
  \[
  \Delta_{\mathbb{T}^2} = -\frac{\partial^2}{\partial t_1^2} - \frac{\partial^2}{\partial t_2^2}.
  \]
- At first sight it seems difficult to construct a differential operator that squares to $\Delta_{\mathbb{T}^2}$:
  \[
  \left( a \frac{\partial}{\partial t_1} + b \frac{\partial}{\partial t_2} \right)^2 = a^2 \frac{\partial^2}{\partial t_1^2} + 2ab \frac{\partial^2}{\partial t_1 \partial t_2} + b^2 \frac{\partial^2}{\partial t_2^2}.
  \]
This puzzle was solved by Dirac who considered the possibility that \( a \) and \( b \) be complex matrices:

\[
a = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}
\]

then \( a^2 = b^2 = -1 \) and \( ab + ba = 0 \).

The Dirac operator on the torus is

\[
D_{\mathbb{T}^2} = \begin{pmatrix} 0 & \frac{\partial}{\partial t_1} + i \frac{\partial}{\partial t_2} \\ -\frac{\partial}{\partial t_1} + i \frac{\partial}{\partial t_2} & 0 \end{pmatrix},
\]

which satisfies \((D_{\mathbb{T}^2})^2 = -\frac{\partial^2}{\partial t_1^2} - \frac{\partial^2}{\partial t_2^2}\).
The 4-dimensional torus

- Consider the 4-torus $\mathbb{T}^4$ parametrized by $t_1, t_2, t_3, t_4$ and the Laplacian is

$$\Delta_{\mathbb{T}^4} = - \frac{\partial^2}{\partial t_1^2} - \frac{\partial^2}{\partial t_2^2} - \frac{\partial^2}{\partial t_3^2} - \frac{\partial^2}{\partial t_4^2}.$$ 

- The search for a differential operator that squares to $\Delta_{\mathbb{T}^4}$ again involves matrices, but we also need quaternions:

$$i^2 = j^2 = k^2 = ijk = -1.$$ 

- The Dirac operator on $\mathbb{T}^4$ is

$$D_{\mathbb{T}^4} = \begin{pmatrix} 0 & \frac{\partial}{\partial t_1} + i \frac{\partial}{\partial t_2} + j \frac{\partial}{\partial t_3} + k \frac{\partial}{\partial t_4} \\ - \frac{\partial}{\partial t_1} + i \frac{\partial}{\partial t_2} + j \frac{\partial}{\partial t_3} + k \frac{\partial}{\partial t_4} & 0 \end{pmatrix}$$

- The relations $ij = -ji$, $ik = -ki$, et cetera imply that its square coincides with $\Delta_{\mathbb{T}^4}$. 

The geometry of $M$ is not fully determined by spectrum of $D_M$.

This is considerably improved by considering besides $D_M$ also the algebra $C^\infty(M)$ of smooth (coordinate) functions on $M$.

In fact, the Riemannian distance function on $M$ is equal to

$$d(x, y) = \sup_{f \in C^\infty(M)} \{|f(x) - f(y)| : \text{gradient } f \leq 1\}$$

The gradient of $f$ is given by the commutator

$$[D_M, f] = D_M f - fD_M$$

(e.g. $[D_{S^1}, f] = -i \frac{df}{dt}$)
Replace spacetime by \textit{spacetime} $\times$ \textit{finite (nc) space}: $M \times F$

- $F$ is considered as \textit{internal space} (Kaluza–Klein like)
- $F$ is described by a \textit{noncommutative algebra}, such as $M_3(\mathbb{C})$, just as spacetime is described by \textit{coordinate functions} $x_\mu(p)$.
- ‘Propagation’ of particles in $F$ is described by a \textit{Dirac-type operator} $D_F$ which is actually simply a hermitian matrix.
Finite spaces

- Finite space $F$, discrete topology

$$F = 1 \bullet 2 \bullet \cdots N \bullet$$

- Smooth functions on $F$ are given by $N$-tuples in $\mathbb{C}^N$, and the corresponding algebra $C^\infty(F)$ corresponds to diagonal matrices

$$
\begin{pmatrix}
 f(1) & 0 & \cdots & 0 \\
 0 & f(2) & \cdots & 0 \\
 \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & \cdots & f(N)
\end{pmatrix}
$$

- The finite Dirac operator is an arbitrary hermitian matrix $D_F$, giving rise to a distance function on $F$ as

$$d(p, q) = \sup_{f \in C^\infty(F)} \{ |f(p) - f(q)| : \| [D_F, f] \| \leq 1 \}$$
Example: two-point space

\[ F = 1 \bullet 2 \bullet \]

- Then the algebra of smooth functions

\[ C^\infty(F) := \left\{ \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \bigg| \lambda_1, \lambda_2 \in \mathbb{C} \right\} \]

- A finite Dirac operator is given by

\[ D_F = \begin{pmatrix} 0 & \overline{c} \\ c & 0 \end{pmatrix} \quad (c \in \mathbb{C}) \]

- The distance formula then becomes

\[ d(1, 2) = \frac{1}{|c|} \]
The geometry of $F$ gets much more interesting if we allow for a noncommutative structure at each point of $F$.

- Instead of diagonal matrices, we consider block diagonal matrices

$$A = \begin{pmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_N \end{pmatrix},$$

where the $a_1, a_2, \ldots, a_N$ are square matrices of size $n_1, n_2, \ldots, n_N$.

- Hence we will consider the matrix algebra

$$A_F := M_{n_1}(\mathbb{C}) \oplus M_{n_2}(\mathbb{C}) \oplus \cdots \oplus M_{n_N}(\mathbb{C})$$

- A finite Dirac operator is still given by a hermitian matrix.
The two-point space can be given a noncommutative structure by considering the algebra $\mathcal{A}_F$ of $3 \times 3$ block diagonal matrices of the following form

$$
\begin{pmatrix}
\lambda & 0 & 0 \\
0 & a_{11} & a_{12} \\
0 & a_{21} & a_{22}
\end{pmatrix}
$$

A finite Dirac operator for this example is given by a hermitian $3 \times 3$ matrix, for example

$$
D_F = 
\begin{pmatrix}
0 & \overline{c} & 0 \\
c & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$
Spectral triples
Noncommutative Riemannian spin manifolds

\((\mathcal{A}, \mathcal{H}, D)\)

- Extended to real spectral triple:
  - \(J : \mathcal{H} \to \mathcal{H}\) real structure (charge conjugation) such that
  \[J^2 = \pm 1; \quad JD = \pm DJ\]
- Right action of \(\mathcal{A}\) on \(\mathcal{H}\):
  \(a^{\text{op}} = Ja^* J^{-1}\) so that
  \((ab)^{\text{op}} = b^{\text{op}} a^{\text{op}}\) and
  \[\left[a^{\text{op}}, b\right] = 0; \quad a, b \in \mathcal{A}\]
- \(D\) is said to satisfy first-order condition if
  \[\left[[D, a], b^{\text{op}}\right] = 0\]
Trace $f(D/\Lambda) + \frac{1}{2} \langle J\tilde{\psi}, D\tilde{\psi} \rangle$

- **Invariant** under unitaries $u \in U(\mathcal{A})$ acting as
  \[ D \mapsto UDU^*; \quad U = u(u^*)^{op} \]

- **Gauge group:** $G(\mathcal{A}) := \{ u(u^*)^{op} : u \in U(\mathcal{A}) \}$.

- **Compute rhs:**
  \[ D \mapsto D + u[D, u^*] \pm Ju[D, u^*]J^{-1} \]
Extend this to more general perturbations:

\[
\text{Pert}(\mathcal{A}) := \left\{ \sum_j a_j \otimes b_j^{\text{op}} \in \mathcal{A} \otimes \mathcal{A}^{\text{op}} \left| \begin{array}{c}
\sum_j a_j b_j = 1 \\
\sum_j a_j \otimes b_j^{\text{op}} = \sum_j b_j^{*} \otimes a_j^{*\text{op}}
\end{array} \right. \right\}
\]

with semi-group law inherited from product in \( \mathcal{A} \otimes \mathcal{A}^{\text{op}} \).

- \( \mathcal{U}(\mathcal{A}) \) maps to \( \text{Pert}(\mathcal{A}) \) by sending \( u \mapsto u \otimes u^{*\text{op}} \).
- \( \text{Pert}(\mathcal{A}) \) acts on \( \mathcal{D} \):

\[
\mathcal{D} \mapsto \sum_j a_j Db_j = \mathcal{D} + \sum_j a_j[D, b_j]
\]

and this also extends to real spectral triples via the map

\[
\text{Pert}(\mathcal{A}) \rightarrow \text{Pert}(\mathcal{A} \otimes J\mathcal{A}J^{-1})
\]
Proposition

Let \( A_F \) be the algebra of block diagonal matrices (fixed size). Then the perturbation semigroup of \( A_F \) is

\[
\text{Pert}(A_F) \cong \left\{ \sum_j A_j \otimes B_j \in A_F \otimes A_F \left| \begin{array}{l}
\sum_j A_j(B_j)^t = I \\
\sum_j A_j \otimes B_j = \sum_j B_j \otimes A_j
\end{array} \right. \right\}
\]

The semigroup law in \( \text{Pert}(A_F) \) is given by the matrix product in \( A_F \otimes A_F \):

\[(A \otimes B)(A' \otimes B') = (AA') \otimes (BB').\]
Example: perturbation semigroup of two-point space

- Now \( \mathcal{A}_F = \mathbb{C}^2 \), the algebra of diagonal \( 2 \times 2 \) matrices.
- In terms of the standard basis of such matrices

\[
e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}
\]

we can write an arbitrary element of \( \text{Pert}(\mathbb{C}^2) \) as

\[
z_1 e_{11} \otimes e_{11} + z_2 e_{11} \otimes e_{22} + z_3 e_{22} \otimes e_{11} + z_4 e_{22} \otimes e_{22}
\]

- Matrix multiplying \( e_{11} \) and \( e_{22} \) yields for the normalization condition:

\[
z_1 = 1 = z_4.
\]

- The self-adjointness condition reads

\[
z_2 = z_3
\]

leaving only one free complex parameter so that \( \text{Pert}(\mathbb{C}^2) \cong \mathbb{C} \).

- More generally, \( \text{Pert}(\mathbb{C}^N) \cong \mathbb{C}^{N(N-1)/2} \) with componentwise product.
Example: perturbation semigroup of $M_2(\mathbb{C})$

- Let us consider a noncommutative example, $A_F = M_2(\mathbb{C})$.
- We can identify $M_2(\mathbb{C}) \otimes M_2(\mathbb{C})$ with $M_4(\mathbb{C})$ so that elements in $\text{Pert}(M_2(\mathbb{C}))$ are $4 \times 4$-matrices satisfying the normalization and self-adjointness condition. In a suitable basis:

$$\text{Pert}(M_2(\mathbb{C})) = \left\{ \begin{pmatrix} 1 & v_1 & v_2 & iv_3 \\ 0 & x_1 & x_2 & ix_3 \\ 0 & x_4 & x_5 & ix_6 \\ 0 & ix_7 & ix_8 & x_9 \end{pmatrix} \bigg| \begin{array}{c} v_1, v_2, v_3 \in \mathbb{R} \\ x_1, \ldots, x_9 \in \mathbb{R} \end{array} \right\}$$

and one can show that

$$\text{Pert}(M_2(\mathbb{C})) \simeq \mathbb{R}^3 \rtimes S.$$

- More generally (B.Sc. thesis Niels Neumann),

$$\text{Pert}(M_N(\mathbb{C})) \simeq W \rtimes S'.$$
Consider noncommutative two-point space described by $\mathbb{C} \oplus M_2(\mathbb{C})$

It turns out that

$$\text{Pert}(\mathbb{C} \oplus M_2(\mathbb{C})) \simeq M_2(\mathbb{C}) \times \text{Pert}(M_2(\mathbb{C}))$$

Only $M_2(\mathbb{C}) \subset \text{Pert}(\mathbb{C} \oplus M_2(\mathbb{C}))$ acts non-trivially on $D_F$:

$$D_F = \begin{pmatrix} 0 & \bar{c} & 0 \\ c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & \bar{c}\phi_1 & \bar{c}\phi_2 \\ c\phi_1 & 0 & 0 \\ c\phi_2 & 0 & 0 \end{pmatrix}$$

Physicists call $\phi_1$ and $\phi_2$ the Higgs field.

The group of unitary block diagonal matrices is now $U(1) \times U(2)$ and an element $(\lambda, u)$ therein acts as

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \mapsto \bar{\lambda}u \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$
Example: perturbation semigroup of a manifold

Recall, for any involutive algebra $A$

\[
\text{Pert}(A) := \left\{ \sum_j a_j \otimes b_j^{\text{op}} \in A \otimes A^{\text{op}} \mid \sum_j a_j b_j = 1, \sum_j a_j \otimes b_j^{\text{op}} = \sum_j b_j^* \otimes a_j^{*\text{op}} \right\}
\]

- We can consider functions in $C^\infty(M) \otimes C^\infty(M)$ as functions of two variables in $C^\infty(M \times M)$.
- The normalization and self-adjointness condition in \( \text{Pert}(C^\infty(M)) \) translate accordingly and yield

\[
\text{Pert}(C^\infty(M)) = \left\{ f \in C^\infty(M \times M) \mid f(x, x) = 1, f(x, y) = f(y, x) \right\}
\]

- The action of \( \text{Pert}(C^\infty(M)) \) on the partial derivatives appearing in a Dirac operator $D_M$ is given by

\[
\frac{\partial}{\partial x_\mu} \mapsto \frac{\partial}{\partial x_\mu} + \frac{\partial}{\partial y_\mu} f(x, y) \bigg|_{y=x} =: \partial_\mu + A_\mu
\]
Applications to particle physics

- Combine (4d) Riemannian spin manifold $M$ with finite noncommutative space $F$:

$$M \times F$$

- $F$ is internal space at each point of $M$

- Described by matrix-valued functions on $M$: algebra $C^\infty(M, A_F)$
Dirac operator on $M \times F$

- Recall the form of $D_M$:

$$D_M = \begin{pmatrix} 0 & D_M^+ \\ D_M^- & 0 \end{pmatrix}. $$

- Dirac operator on $M \times F$ is the combination

$$D_{M \times F} = D_M + \gamma_5 D_F = \begin{pmatrix} D_F & D_M^+ \\ D_M^- & -D_F \end{pmatrix}. $$

- The crucial property of this specific form is that it squares to the sum of the two Laplacians on $M$ and $F$:

$$D_{M \times F}^2 = D_M^2 + D_F^2 $$

- Using this, we can expand the heat trace:

$$\text{Trace } e^{-D_{M \times F}^2 / \Lambda^2} = \frac{\text{Vol}(M)\Lambda^4}{(4\pi)^2} \text{Trace} \left( 1 - \frac{D_F^2}{\Lambda^2} + \frac{D_F^4}{2\Lambda^4} \right) + \mathcal{O}(\Lambda^{-1}).$$
The Higgs mechanism

We apply this to the noncommutative two-point space described before

- **Algebra** $\mathcal{A}_F = \mathbb{C} \oplus M_2(\mathbb{C})$
- **Perturbation** of Dirac operator $D_M$ parametrized by gauge bosons for $U(1) \times U(2)$.
- **Perturbation** of finite Dirac operator $D_F$ parametrized by $\phi_1, \phi_2$.
- **Spectral action** for the perturbed Dirac operator induces a potential:

$$V(\phi) = -2\Lambda^2(|\phi_1|^2 + |\phi_2|^2) + (|\phi_1|^2 + |\phi_2|^2)^2$$
Describe $M \times F_{SM}$ by [CCM 2007]

- **Coordinates:** $\hat{x}^\mu(p) \in \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ (with unimodular unitaries $U(1)_Y \times SU(2)_L \times SU(3)$).
- **Dirac operator** $D_{M\times F} = D_M + \gamma_5 D_F$ where

$$D_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$$

is a $96 \times 96$-dimensional hermitian matrix where 96 is:

$$3 \times 2 \times ( 2 \otimes 1 + 1 \otimes 1 + 1 \otimes 1 + 2 \otimes 3 + 1 \otimes 3 + 1 \otimes 3 )$$

The Dirac operator on $F_{\text{SM}}$

$$D_F = \begin{pmatrix} S & T^* \\ T & S \end{pmatrix}$$

- The operator $S$ is given by

$$S_l := \begin{pmatrix} 0 & 0 & Y_\nu & 0 \\ 0 & 0 & 0 & Y_e \\ Y_\nu^* & 0 & 0 & 0 \\ 0 & Y_e^* & 0 & 0 \end{pmatrix}, \quad S_q \otimes 1_3 = \begin{pmatrix} 0 & 0 & Y_u & 0 \\ 0 & 0 & 0 & Y_d \\ Y_u^* & 0 & 0 & 0 \\ 0 & Y_d^* & 0 & 0 \end{pmatrix} \otimes 1_3,$$

where $Y_\nu$, $Y_e$, $Y_u$ and $Y_d$ are $3 \times 3$ mass matrices acting on the three generations.

- The symmetric operator $T$ only acts on the right-handed (anti)neutrinos, $T \nu_R = Y_R \bar{\nu}_R$ for a $3 \times 3$ symmetric Majorana mass matrix $Y_R$, and $Tf = 0$ for all other fermions $f \neq \nu_R$. 
Inner perturbations

• Inner perturbations of $D_M$ give a matrix

$$A_\mu = \begin{pmatrix} B_\mu & 0 & 0 & 0 \\ 0 & W^{3}_\mu & W^{+}_\mu & 0 \\ 0 & W^{-}_\mu & -W^{3}_\mu & 0 \\ 0 & 0 & 0 & (G^{a}_\mu) \end{pmatrix}$$

corresponding to hypercharge, weak and strong interaction.

• Inner perturbations of $D_F$ give

$$\begin{pmatrix} Y_\nu & 0 \\ 0 & Y_e \end{pmatrix} \leadsto \begin{pmatrix} Y_\nu \phi_1 & -Y_e \phi_2 \\ Y_\nu \phi_2 & Y_e \phi_1 \end{pmatrix}$$

corresponding to SM-Higgs field. Similarly for $Y_u, Y_d$. 
If we consider the spectral action:

\[
\text{Trace } f(D_M/\Lambda) \sim c_0 \int F_{\mu\nu} F^{\mu\nu} - c_2' |\phi|^2 + c_0' |\phi|^4 + \cdots
\]

we observe [CCM 2007]:

- The coupling constants of hypercharge, weak and strong interaction are expressed in terms of the single constant \(c_0\) which implies

\[
g_3^2 = g_2^2 = \frac{5}{3} g_1^2
\]

In other words, there should be grand unification.

- Moreover, the quartic Higgs coupling \(\lambda\) is related via

\[
\lambda \approx 24 \frac{3 + \rho^4}{(3 + \rho^2)^2} g_2^2; \quad \rho = \frac{m_\nu}{m_{\text{top}}}
\]

\[\text{Dynamics and interactions}\]
This can be used to derive predictions as follows:

- Interpret the spectral action as an **effective field theory** at $\Lambda_{\text{GUT}} \approx 10^{13} - 10^{16}$ GeV.
- Run the quartic coupling constant $\lambda$ to SM-energies to predict

\[
m_h^2 = \frac{4\lambda M_W^2}{3g_2^2}
\]

This gives [CCM 2007]

\[
167 \text{ GeV} \leq m_h \leq 176 \text{ GeV}
\]
Three problems

1. This prediction is **falsified** by the now measured value.

2. In the Standard Model there is not the presumed grand unification.

3. There is a problem with the low value of $m_h$, making the Higgs vacuum un/metastable [Elias-Miro et al. 2011].
Beyond the SM with noncommutative geometry
A solution to the above three problems?

• The matrix coordinates of the Standard Model arise naturally as a restriction of the following coordinates

\[ \hat{x}^\mu(p) = (q^\mu_R(p), q^\mu_L(p), m^\mu(p)) \in \mathbb{H}_R \oplus \mathbb{H}_L \oplus M_4(\mathbb{C}) \]

corresponding to a Pati–Salam unification:

\[ U(1)_Y \times SU(2)_L \times SU(3) \rightarrow SU(2)_R \times SU(2)_L \times SU(4) \]

• The 96 fermionic degrees of freedom are structured as

\[
\begin{pmatrix}
\nu_R & u_{iR} \\
e_R & d_{iR}
\end{pmatrix}
\begin{pmatrix}
\nu_L & u_{iL} \\
e_L & d_{iL}
\end{pmatrix}
(i = 1, 2, 3)
\]

• Again the finite Dirac operator is a 96 × 96-dimensional matrix (details in [CCS 2013]).
Inner perturbations

- Inner perturbations of $D_M$ now give three gauge bosons:

$$W_R^\mu, \quad W_L^\mu, \quad V^\mu$$

Corresponding to $SU(2)_R \times SU(2)_L \times SU(4)$.

- For the inner perturbations of $D_F$ we distinguish two cases, depending on the initial form of $D_F$:
  1. The Standard Model $D_F = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}$
  2. A more general $D_F$ with zero $\bar{f}_L - f_L$-interactions.
Scalar sector of the spectral Pati–Salam model

Case I For a SM $D_F$, the resulting scalar fields are composite fields, expressed in scalar fields whose representations are:

$$
\begin{array}{c|ccc}
\phi^b_a & SU(2)_R & SU(2)_L & SU(4) \\
\Delta_{\dot{a}I} & 2 & 2 & 1 \\
\Sigma_{IJ}^I & 2 & 1 & 4 \\
\Sigma_{IJ}^I & 1 & 1 & 15 \\
\end{array}
$$

Case II For a more general finite Dirac operator, we have fundamental scalar fields:

$$
\begin{array}{c|ccc}
\text{particle} & SU(2)_R & SU(2)_L & SU(4) \\
\Sigma^{bJ}_{\dot{a}J} & 2 & 2 & 1 + 15 \\
H_{\dot{a}IbJ} & 3 & 1 & 10 \\
& 1 & 1 & 6 \\
\end{array}
$$
As for the Standard Model, we can compute the spectral action which describes the usual Pati–Salam model with

- unification of the gauge couplings

\[ g_R = g_L = g. \]

- A rather involved, fixed scalar potential, still subject to further study
However, independently from the spectral action, we can analyze the running at one loop of the gauge couplings [CCS 2015]:

1. We run the **Standard Model gauge couplings** up to a presumed PS → SM symmetry breaking scale $m_R$

2. We take their values as **boundary conditions** to the Pati–Salam gauge couplings $g_R, g_L, g$ at this scale via

   \[
   \frac{1}{g_1^2} = \frac{2}{3} \frac{1}{g^2} + \frac{1}{g_R^2}, \quad \frac{1}{g_2^2} = \frac{1}{g_L^2}, \quad \frac{1}{g_3^2} = \frac{1}{g^2},
   \]

3. Vary $m_R$ in a search for a **unification scale** $\Lambda$ where

   \[g_R = g_L = g\]

which is where the **spectral action** is valid as an **effective theory**.
For the **Standard Model Dirac operator**, we have found that with $m_R \approx 4.25 \times 10^{13}$ GeV there is unification at $\Lambda \approx 2.5 \times 10^{15}$ GeV:
In this case, we can also say something about the scalar particles that remain after SSB:

\[
\begin{align*}
\begin{pmatrix}
\phi^0_1 \\
\phi^-_1 \\
\phi^0_2 \\
\phi^-_2
\end{pmatrix}
&=
\begin{pmatrix}
\phi^1_1 \\
\phi^1_2
\end{pmatrix} \\
\begin{pmatrix}
\phi^-_2 \\
\phi^0_2
\end{pmatrix}
&=
\begin{pmatrix}
\phi^2_1 \\
\phi^2_2
\end{pmatrix}
\end{align*}
\]

\[
\begin{array}{c|ccc}
& U(1)_Y & SU(2)_L & SU(3) \\
\hline
\phi^0_1 & 0 & 1 & 2 \\
\phi^-_1 & -1 & 1 & 2 \\
\phi^0_2 & 0 & 1 & 2 \\
\phi^-_2 & -1 & 1 & 2 \\
\sigma & 0 & 1 & 1 \\
\eta & -1/3 & 1 & 3
\end{array}
\]

- It turns out that these scalar fields have a little influence on the running of the SM-gauge couplings (at one loop).
- However, this sector contains the real scalar singlet $\sigma$ that allowed for a realistic Higgs mass and that stabilizes the Higgs vacuum [CC 2012].
For the more general case, we have found that with \( m_R \approx 1.5 \times 10^{11} \) GeV there is unification at \( \Lambda \approx 6.3 \times 10^{16} \) GeV:
Conclusion

We have arrived at a spectral Pati–Salam model that

- goes beyond the Standard Model
- has a fixed scalar sector once the finite Dirac operator has been fixed (only a few scenarios)
- exhibits grand unification for all of these scenarios (confirmed by [Aydemir–Minic–Sun–Takeuchi 2015])
- the scalar sector has the potential to stabilize the Higgs vacuum and allow for a realistic Higgs mass.
Further reading

A. Chamseddine, A. Connes, WvS.


WvS.


and also: [http://www.noncommutativegeometry.nl](http://www.noncommutativegeometry.nl)