Beyond the spectral Standard Model: Pati–Salam unification

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- Spectral geometry
- Matrix algebra and noncommutative geometry
- Perturbation semigroup: gauge and Higgs fields
- The spectral Standard Model and Beyond: Grand unfication

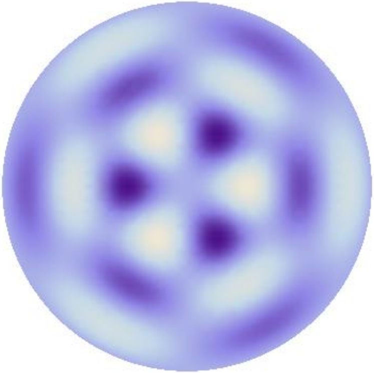
"Can one hear the shape of a drum?" (Kac, 1966)

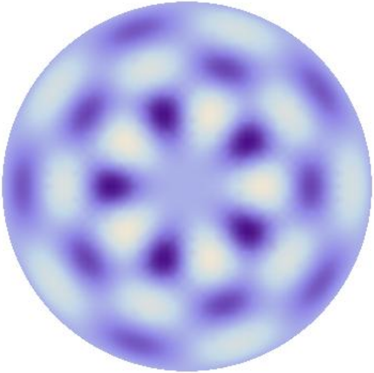
Or, more precisely, given a Riemannian manifold M, does the spectrum of wave numbers k in the Helmholtz equation

$$\boxed{\Delta_M u = k^2 u}$$

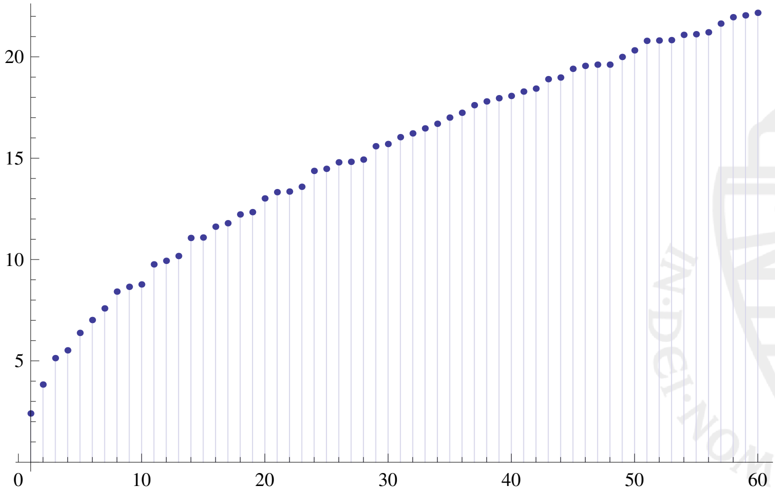
determine the geometry of M?

The disc

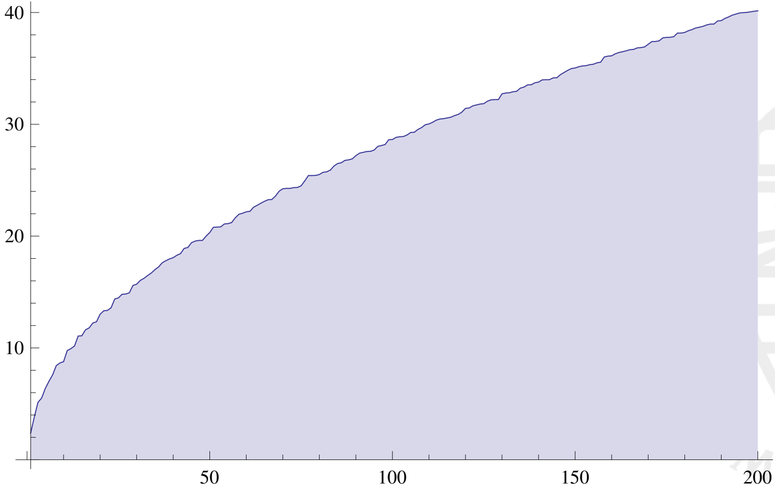




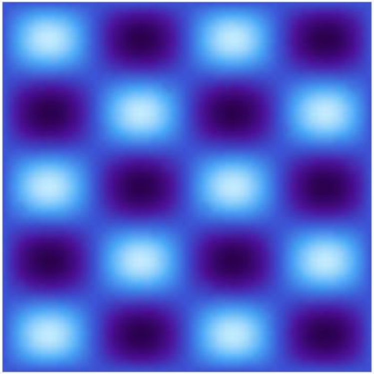
Wave numbers on the disc

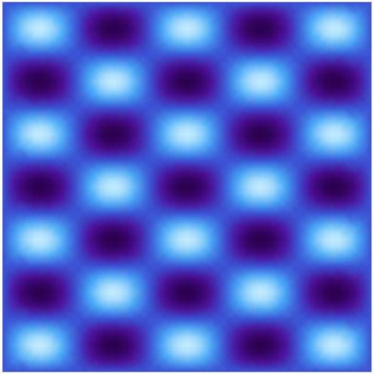


Wave numbers on the disc: high frequencies

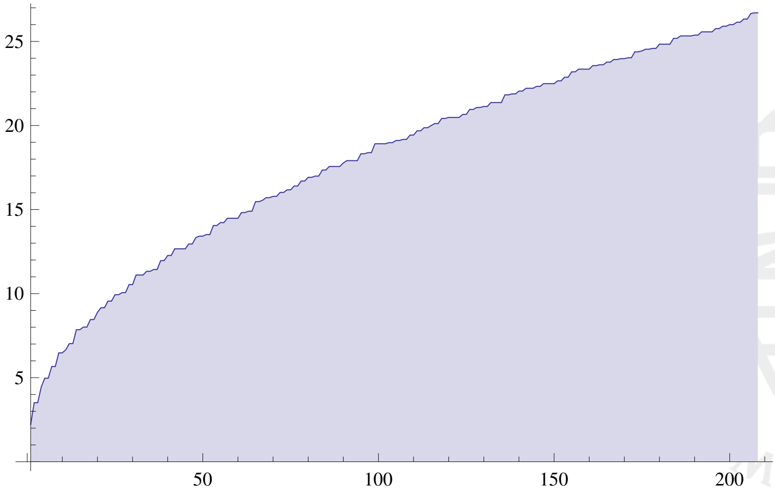


The square



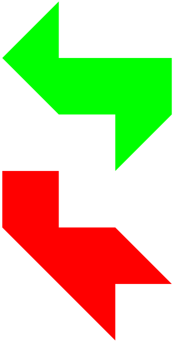


Wave numbers on the square



Isospectral domains

But, there are isospectral domains in $\mathbb{R}^2$:



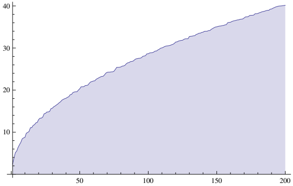
(Gordon, Webb, Wolpert, 1992)

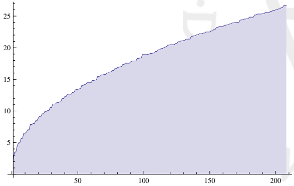
so the answer to Kac's question is no.

Nevertheless, certain information can be extracted from spectrum, such as dimension n of M:

$$N(\Lambda) = \#\text{wave numbers } \leq \Lambda$$
$$\sim \frac{\Omega_n \mathrm{Vol}(M)}{n(2\pi)^n} \Lambda^n$$

For the disc and square this is confirmed by the parabolic shapes ($\sqrt{\Lambda}$):





Recall that k^2 is an eigenvalue of the Laplacian in the Helmholtz equation.

- The Dirac operator D_M is a 'square-root' of the Laplacian, so that its spectrum give the wave numbers k.
- First found by Paul Dirac in flat space, but exists on any Riemannian spin manifold M.

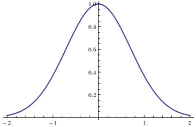
- Reconsider Weyl's estimate, in a smooth version:

$$\mathrm{Tr}\ f\left(\frac{D_M}{\Lambda}\right) = \sum_\lambda f\left(\frac{\lambda}{\Lambda}\right)$$

for a smooth cutoff function $f : \mathbb{R} \to \mathbb{R}$.

- For simplicity, (today) restrict to a Gaussian function

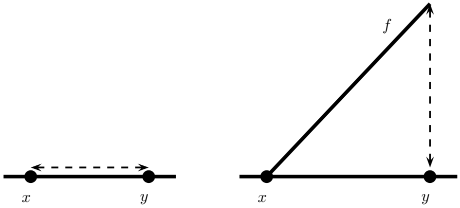
$$f(x) = e^{-x^2}$$



so that we can use heat asymptotics: $\mathrm{Tr}\ e^{-D_M^2/\Lambda^2} \sim \dfrac{\mathrm{Vol}(M)\Lambda^n}{(4\pi)^{n/2}}$

- As said, the geometry of M is not fully determined by spectrum of D_M.
- This can be improved by considering besides D_M also the algebra $C^\infty(M)$ generated by all coordinate functions on M.
- This combination of coordinate algebra and operators is central to the spectral, or noncommutative approach [C 1994].
- Reconstruction of M in the commutative case [C 1989]:

$$d(x,y) = \sup_{f \in C^\infty(M)} \{ |f(x) - f(y)| : \| [D_M, f] \| \leq 1 \}$$



Noncommutative fine structure of spacetime

Replace spacetime by
spacetime $\times$ **noncommutative space**: $M \times F$

- F is considered as finite internal space (Kaluza–Klein like)
- F is described by noncommutative matrices, that play the role of coordinates, just as spacetime is described by $x_\mu(p)$.
- These matrices geometrically encode non-abelian gauge symmetries.
- 'Propagation' of particles in F is described by a 'Dirac operator' D_F which is actually simply a hermitian matrix.

Note that the spectral approach is now the only way to describe the geometry of F.

Finite **commutative** spaces

- Finite space F

$$F = \quad {}_1\bullet \quad {}_2\bullet \quad \cdots\cdots \quad {}_N\bullet$$

- Coordinate functions on F are given by N-tuples in $\mathbb{C}^N$, and the corresponding algebra $C^\infty(F)$ corresponds to diagonal matrices

$$\begin{pmatrix} f(1) & 0 & \cdots & 0 \\ 0 & f(2) & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & f(N) \end{pmatrix}$$

- The finite Dirac operator is an arbitrary hermitian matrix D_F, giving rise to a distance function on F as

$$d(p,q) = \sup_{f \in C^\infty(F)} \{ |f(p) - f(q)| : \| [D_F, f] \| \leq 1 \}$$

Example: two-point space

$$F = {}_1\bullet \qquad {}_2\bullet$$

- Then the algebra of smooth functions

$$C^\infty(F) := \left\{ \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \middle| \lambda_1, \lambda_2 \in \mathbb{C} \right\}$$

- A finite Dirac operator is given by

$$D_F = \begin{pmatrix} 0 & \overline{c} \\ c & 0 \end{pmatrix}; \qquad (c \in \mathbb{C})$$

- The distance formula then becomes

$$d(1,2) = \frac{1}{|c|}$$

Finite **noncommutative** spaces

The geometry of F gets much more interesting if we allow for a *noncommutative* structure at each point of F.

- Instead of diagonal matrices, we consider block diagonal matrices

$$A = \begin{pmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & a_N \end{pmatrix},$$

  where the $a_1, a_2, \dots, a_N$ are square matrices of size $n_1, n_2, \dots, n_N$.

- Hence we will consider the matrix algebra

$$A_F := M_{n_1}(\mathbb{C}) \oplus M_{n_2}(\mathbb{C}) \oplus \cdots \oplus M_{n_N}(\mathbb{C})$$

  where $\mathbb{C}$ can be replaced by $\mathbb{R}$ or $\mathbb{H}$.

- A finite Dirac operator is still given by a hermitian matrix.

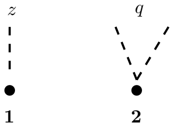
Example: noncommutative two-point space

Coordinates on F are elements in $\mathbb{C} \oplus \mathbb{H}$

- A complex number z
- A quaternion $q = q_0 + iq_k\sigma^k$; in terms of Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

It describes a two-point space, with internal structure:



- Gauge group is given by unitaries: $U(1) \times SU(2)$.
- 'Dirac operator'

$$D_F = \begin{pmatrix} 0 & \overline{c} & 0 \\ c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Mathematical intermezzo: inner perturbations

Chamseddine–Connes-vS (2013)

We make the above more dynamical by *perturbing* D_F by matrices in A_F.

Definition

Let A_F be the above algebra of block diagonal matrices (fixed size). The perturbation semigroup of A_F is defined as

$$\mathrm{Pert}(A_F) := \left\{ \sum_j A_j \otimes B_j \in A_F \otimes A_F \;\middle|\; \begin{array}{l} \sum_j A_j (B_j)^t = \mathbb{I} \\ \sum_j A_j \otimes B_j = \sum_j \overline{B_j} \otimes \overline{A_j} \end{array} \right\}$$

The semigroup law in $\mathrm{Pert}(A_F)$ is given by the matrix product in $A_F \otimes A_F$:

$$(A \otimes B)(A' \otimes B') = (AA') \otimes (BB').$$

Example: perturbation semigroup of two-point space

- Now $A_F = \mathbb{C}^2$, the algebra of diagonal 2×2 matrices.
- In terms of the standard basis of such matrices

$$e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

we can write an arbitrary element of $\mathrm{Pert}(\mathbb{C}^2)$ as

$$z_1 e_{11} \otimes e_{11} + z_2 e_{11} \otimes e_{22} + z_3 e_{22} \otimes e_{11} + z_4 e_{22} \otimes e_{22}$$

- Matrix multiplying e_{11} and e_{22} yields for the normalization condition:

$$z_1 = 1 = z_4.$$

- The self-adjointness condition reads

$$z_2 = \overline{z_3}$$

leaving only one free complex parameter so that $\mathrm{Pert}(\mathbb{C}^2) \simeq \mathbb{C}$.
- More generally, $\mathrm{Pert}(\mathbb{C}^N) \simeq \mathbb{C}^{N(N-1)/2}$ with componentwise product.

Example: perturbation semigroup of $M_N(\mathbb{C})$
with Niels Neumann (B.Sc.)

- Let us consider a noncommutative example, $A_F = M_N(\mathbb{C})$.
- We compute that

$$\mathrm{Pert}(M_N(\mathbb{C})) \cong \left\{ \begin{pmatrix} 1 & v & iw \\ 0 & C & iD \\ 0 & iE & F \end{pmatrix} : v, w, C, D, E, F \quad \text{real-valued} \right\}$$

- This is compatible with the decomposition $\mathbb{C}^N \otimes \overline{\mathbb{C}^N} \cong \mathbb{C} \oplus \mathbb{C}^{N^2-1}$ into irreps of $U(N)$.
- Similar decompositions can be shown to hold for $\mathrm{Pert}(M_N(\mathbb{R}))$ and irreps of $O(N)$, and $\mathrm{Pert}(M_N(\mathbb{H}))$ and irreps of $Sp(N)$.
- Moreover, for direct sums we have

$$\mathrm{Pert}(A \oplus B) \cong \mathrm{Pert}(A) \times \mathrm{Pert}(B) \times (A \otimes B \oplus B \otimes A)^{\mathrm{s.a.}}$$

Example: perturbation semigroup of a manifold

- The perturbation semigroup can be defined for any involutive unital associative algebra A, in particular for $C^\infty(M)$.
- We can consider functions in the tensor product $C^\infty(M) \otimes C^\infty(M)$ as functions of two-variables, *i.e.* elements in $C^\infty(M \times M)$.
- The normalization and self-adjointness condition in $\mathrm{Pert}(C^\infty(M))$ translate accordingly and yield

$$\mathrm{Pert}(C^\infty(M)) = \left\{ f \in C^\infty(M \times M) \,\middle|\, \begin{array}{l} f(x,x) = 1 \\ f(x,y) = \overline{f(y,x)} \end{array} \right\},$$

Action of $\mathrm{Pert}(A_F)$ on Dirac operators

- Action of $\mathrm{Pert}(A_F)$ on hermitian matrices D_F:

$$D_F \mapsto \sum_j A_j D_F B_j^t = D_F + \sum_j A_j [D_F, B_j^t]$$

- This induces a semigroup structure on 'gauge fields'
- The unitary block diagonal matrices $U(A_F)$ in A_F forms the 'gauge group' which is a subgroup of the semigroup $\mathrm{Pert}(A_F)$ via $U \mapsto U \otimes \overline{U}$.
- The restriction of the above action to the unitary group $\mathcal{U}(A_F)$ gives

$$D_F \mapsto U D_F U^* = D_F + U[D_F, U^*]$$

Perturbations on noncommutative two-point space

- Consider noncommutative two-point space described by $\mathbb{C} \oplus M_2(\mathbb{C})$:

$$\mathrm{Pert}(\mathbb{C} \oplus M_2(\mathbb{C})) \simeq M_2(\mathbb{C}) \times \mathrm{Pert}(M_2(\mathbb{C}))$$

- Action of $\mathrm{Pert}(\mathbb{C} \oplus M_2(\mathbb{C}))$ on D_F:

$$D_F = \begin{pmatrix} 0 & \overline{c} & 0 \\ c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & \overline{c}\overline{\phi_1} & \overline{c}\overline{\phi_2} \\ c\phi_1 & 0 & 0 \\ c\phi_2 & 0 & 0 \end{pmatrix}$$

- We may call ϕ_1 and ϕ_2 the Higgs field.
- Indeed, the group of unitary block diagonal matrices is now $U(1) \times U(2)$ and an element (λ, u) therein acts as

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \mapsto \overline{\lambda} u \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$

- The action of $\mathrm{Pert}(C^\infty(M))$ on the partial derivatives appearing in a Dirac operator D_M is given by

$$\frac{\partial}{\partial x_\mu} \mapsto \frac{\partial}{\partial x_\mu} + \frac{\partial}{\partial y_\mu} f(x,y)\Big|_{y=x}; \qquad (\mu = 1\dots,n),$$

where $f \in C^\infty(M \times M)$ is such that $f(x,x) = 1$ and $\overline{f(x,y)} = f(y,x)$.

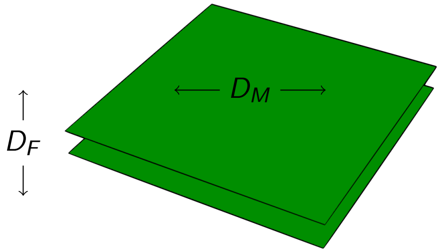
- In physics, one writes

$$A_\mu := \frac{\partial}{\partial y_\mu} f(x,y)\Big|_{y=x}$$

which turns out to be the electromagnetic potential

... end of mathematical intermezzo...

Almost-commutative spacetimes



We now combine mild matrix noncommutativity with spacetime:

- coordinates of the almost-commutative spacetime $M \times F$:

$$\boxed{\hat{x}^\mu(p) = (z^\mu(p), q^\mu(p))}$$

  as elements in $\mathbb{C} \oplus \mathbb{H}$ (for each μ and each point p of M)
- The combined Dirac operator becomes

$$\boxed{D_{M\times F} = D_M + \gamma_5 D_F}$$

  Note that $D^2_{M\times F} = D^2_M + D^2_F$, which will be useful later on.

Inner perturbations on $M \times F$

So, we describe $M \times F$ by:

$$\hat{x}^\mu = (z^\mu, q^\mu)\,; \qquad D_{M\times F} = D_M + \gamma_5 D_F$$

As before, we consider inner perturbations of $D_{M\times F}$ by $\hat{x}^\mu(p)$:

- The inner perturbations of D_F become scalar fields ϕ_1, ϕ_2.
- The inner perturbations of D_M become matrix-valued:

$$\sum_j a_j[D_M, a_j'] = a_\nu \gamma^\mu (\partial_\mu \hat{x}^\nu) =: D_M + A_\mu \gamma^\mu$$

with A_μ taking values in $\mathbb{C} \oplus \mathbb{H}$:

$$A_\mu = \begin{pmatrix} B_\mu & 0 & 0 \\ 0 & W_\mu^3 & W_\mu^+ \\ 0 & W_\mu^- & -W_\mu^3 \end{pmatrix}$$

corresponding to hypercharge and the W-bosons.

Action functional: electroweak theory

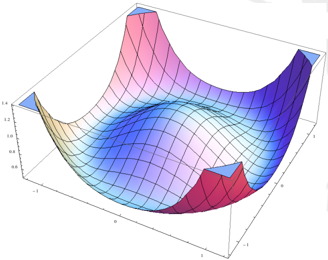
Use $D^2_{M\times F} = D^2_M + D^2_F$ to compute the spectral action

$$\mathrm{Tr}\ e^{-D^2_{M\times F}/\Lambda^2} = \mathrm{Tr}\ e^{-D^2_M/\Lambda^2}\left(1 - \frac{D_F^2}{\Lambda^2} + \frac{1}{2}\frac{D_F^4}{\Lambda^4} - \cdots\right)$$

$$\sim \left(c_4\Lambda^4 \mathrm{Vol}(M) + c_2\Lambda^2\int R\sqrt{g} + c_0\int F_{\mu\nu}F^{\mu\nu}\right)\left(1 - \frac{|\phi|^2}{\Lambda^2} + \frac{|\phi|^4}{2\Lambda^4}\right) + \cdots$$

We now recognize in terms of the field-strength $F_{\mu\nu}$ for A_μ:

- The Yang–Mills term $F_{\mu\nu}F^{\mu\nu}$ for hypercharge and W-boson
- The Higgs potential $-\mu^2|\phi|^2 + \lambda|\phi|^4$



Describe $M \times F_{SM}$ by [CCM 2007]

- Coordinates: $\hat{x}^\mu(p) \in \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ (with unimodular unitaries $U(1)_Y \times SU(2)_L \times SU(3)$).
- Dirac operator $D_{M\times F} = D_M + \gamma_5 D_F$ where

$$D_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$$

is a 96×96-dimensional hermitian matrix where 96 is:

$$3 \times 2 \times (\underline{2} \otimes \underline{1} + \underline{1} \otimes \underline{1} + \underline{1} \otimes \underline{1} + \underline{2} \otimes \underline{3} + \underline{1} \otimes \underline{3} + \underline{1} \otimes \underline{3})$$

families (3), anti-particles (2)

(ν_L, e_L) $\quad \nu_R$ $\quad e_R$ $\quad (u_L, d_L)$ $\quad u_R$ $\quad d_R$

$$D_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$$

- The operator S is given by

$$S_l := \begin{pmatrix} 0 & 0 & Y_\nu & 0 \\ 0 & 0 & 0 & Y_e \\ Y_\nu^* & 0 & 0 & 0 \\ 0 & Y_e^* & 0 & 0 \end{pmatrix}, \qquad S_q \otimes 1_3 = \begin{pmatrix} 0 & 0 & Y_u & 0 \\ 0 & 0 & 0 & Y_d \\ Y_u^* & 0 & 0 & 0 \\ 0 & Y_d^* & 0 & 0 \end{pmatrix} \otimes 1_3,$$

  where Y_ν, Y_e, Y_u and Y_d are 3×3 mass matrices acting on the three generations.
- The symmetric operator T only acts on the right-handed (anti)neutrinos, $T\nu_R = Y_R\overline{\nu_R}$ for a 3×3 symmetric Majorana mass matrix Y_R, and $Tf = 0$ for all other fermions $f \neq \nu_R$.

Inner perturbations

Just as before, we find

- Inner perturbations of D_M give a matrix

$$A_\mu = \begin{pmatrix} B_\mu & 0 & 0 & 0 \\ 0 & W_\mu^3 & W_\mu^+ & 0 \\ 0 & W_\mu^- & -W_\mu^3 & 0 \\ 0 & 0 & 0 & (G_\mu^a) \end{pmatrix}$$

  corresponding to hypercharge, weak and strong interaction.

- Inner perturbations of D_F give

$$\begin{pmatrix} Y_\nu & 0 \\ 0 & Y_e \end{pmatrix} \rightsquigarrow \begin{pmatrix} Y_\nu \phi_1 & -Y_e \overline{\phi}_2 \\ Y_\nu \phi_2 & Y_e \overline{\phi}_1 \end{pmatrix}$$

  corresponding to SM-Higgs field. Similarly for Y_u, Y_d.

Dynamics and interactions

If we reconsider the spectral action:

$$\mathrm{Tr}\ e^{-D_{M\times F}^2/\Lambda^2} \sim \left(c_4 \Lambda^4 \mathrm{Vol}(M) + c_0 \int F_{\mu\nu}F^{\mu\nu} \right) \left(1 - \frac{|\phi|^2}{\Lambda^2} + \frac{|\phi|^4}{2\Lambda^4} \right) + \cdots$$

we observe [CCM 2007]:

- The coupling constants of hypercharge, weak and strong interaction are expressed in terms of the single constant c_0 which implies

$$\boxed{g_3^2 = g_2^2 = \frac{5}{3} g_1^2}$$

  In other words, there should be grand unification.

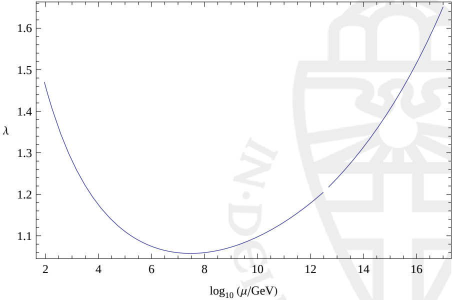
- Moreover, the quartic Higgs coupling λ is related via

$$\lambda \approx 24 \frac{3+\rho^4}{(3+\rho^2)^2} g_2^2; \qquad \rho = \frac{m_\nu}{m_{\mathrm{top}}}$$

This can be used to derive predictions as follows:

- Interpret the spectral action as an effective field theory at $\Lambda_{\mathrm{GUT}} \approx 10^{13} - 10^{16}$ GeV.
- Run the quartic coupling constant λ to SM-energies to predict

$$m_h^2 = \frac{4\lambda M_W^2}{3g_2^2}$$

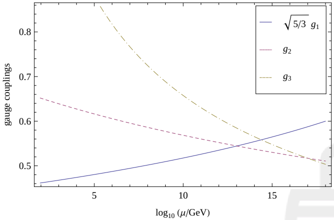


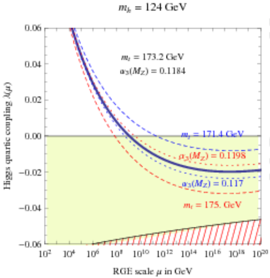
This gives [CCM 2007]

$$\boxed{167 \text{ GeV} \leq m_h \leq 176 \text{ GeV}}$$

Three problems

1. This prediction is falsified by the now measured value.
2. In the Standard Model there is not the presumed grand unification.
3. There is a problem with the low value of m_h, making the Higgs vacuum un/metastable [Elias-Miro et al. 2011].





- The matrix coordinates of the Standard Model arise naturally as a restriction of the following coordinates

$$\hat{x}^\mu(p) = \left(q_R^\mu(p), q_L^\mu(p), m^\mu(p)\right) \in \mathbb{H}_R \oplus \mathbb{H}_L \oplus M_4(\mathbb{C})$$

corresponding to a Pati–Salam unification:

$$U(1)_Y \times SU(2)_L \times SU(3) \to SU(2)_R \times SU(2)_L \times SU(4)$$

- The 96 fermionic degrees of freedom are structured as

$$\left(\begin{array}{cc|cc} \nu_R & u_{iR} & \nu_L & u_{iL} \\ e_R & d_{iR} & e_L & d_{iL} \end{array}\right) \qquad (i = 1, 2, 3)$$

- Again the finite Dirac operator is a 96×96-dimensional matrix (details in [CCS 2013]).

- Inner perturbations of D_M now give three gauge bosons:

$$W_R^\mu, \qquad W_L^\mu, \qquad V^\mu$$

corresponding to $SU(2)_R \times SU(2)_L \times SU(4)$.
- For the inner perturbations of D_F we distinguish two cases, depending on the initial form of D_F:
  I The Standard Model $D_F = \begin{pmatrix} S & T^* \\ T & \overline{S} \end{pmatrix}$
  II A more general D_F with zero $\overline{f}_L - f_L$-interactions.

Scalar sector of the spectral Pati–Salam model

- For a SM D_F, the resulting scalar fields are composite fields, expressed in scalar fields whose representations are:

	$SU(2)_R$	$SU(2)_L$	$SU(4)$
$\phi_{\dot{a}}^{b}$	2	2	1
$\Delta_{\dot{a}I}$	2	1	4
Σ_J^I	1	1	15

- For a more general finite Dirac operator, we have fundamental scalar fields:

particle	$SU(2)_R$	$SU(2)_L$	$SU(4)$
$\Sigma_{\dot{a}J}^{bJ}$	2	2	$1+15$
$H_{\dot{a}I\dot{b}J}$	3	1	10
	1	1	6

Action functional

As for the Standard Model, we can compute the spectral action which describes the usual Pati–Salam model with

- unification of the gauge couplings

$$g_R = g_L = g.$$

- A rather involved, fixed scalar potential, still subject to further study

However, independently from the spectral action, we can analyze the running at one loop of the gauge couplings [CCS 2015, AMST 2015]:

1. At Λ there is a unification of the Pati–Salam gauge couplings

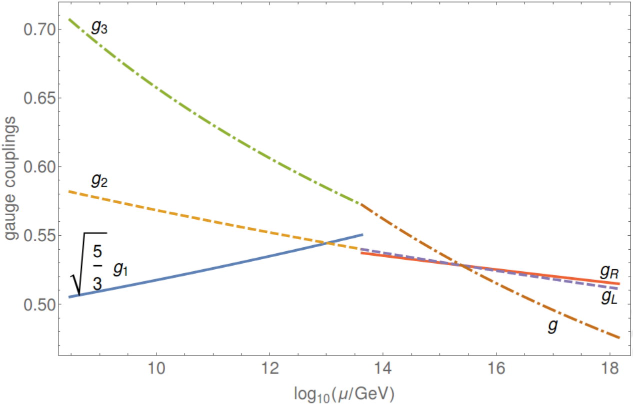
$$g_R = g_L = g$$

   This is where the spectral action is valid as an effective theory.
2. Run the Pati–Salam gauge couplings down to a presumed PS $\to$ SM symmetry breaking scale m_R
3. Relate their values to the Standard Model gauge couplings at this scale via

$$\frac{1}{g_1^2} = \frac{2}{3}\frac{1}{g^2} + \frac{1}{g_R^2}, \qquad \frac{1}{g_2^2} = \frac{1}{g_L^2}, \qquad \frac{1}{g_3^2} = \frac{1}{g^2},$$

   and run couplings down to scale M_Z.
4. Look for values of m_R and Λ that yield the measured values of g_1, g_2, g_3 at energy scale M_Z.

For the Standard Model Dirac operator, we have found that with $m_R \approx 4.25 \times 10^{13}$ GeV there can be unification at $\Lambda \approx 2.5 \times 10^{15}$ GeV:



In this case, we can also say something about the scalar particles that remain after SSB:

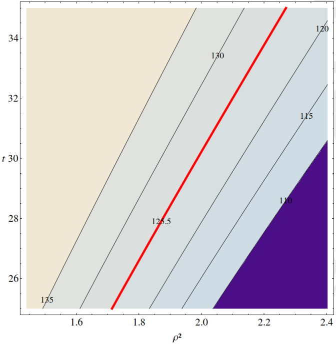
	$U(1)_Y$	$SU(2)_L$	$SU(3)$
$\begin{pmatrix}\phi_1^0\\ \phi_1^+\end{pmatrix} = \begin{pmatrix}\phi_{\dot{1}}^1\\ \phi_{\dot{1}}^2\end{pmatrix}$	1	2	1
$\begin{pmatrix}\phi_2^-\\ \phi_2^0\end{pmatrix} = \begin{pmatrix}\phi_{\dot{2}}^1\\ \phi_{\dot{2}}^2\end{pmatrix}$	-1	2	1
σ	0	1	1
η	$-\frac{2}{3}$	1	3

- It turns out that these scalar fields have a little influence on the running of the SM-gauge couplings (at one loop).
- However, this sector contains the real scalar singlet σ of [CC 2012].

- Suppose that the real scalar singlet σ is coupled to the Higgs sector in the following way:

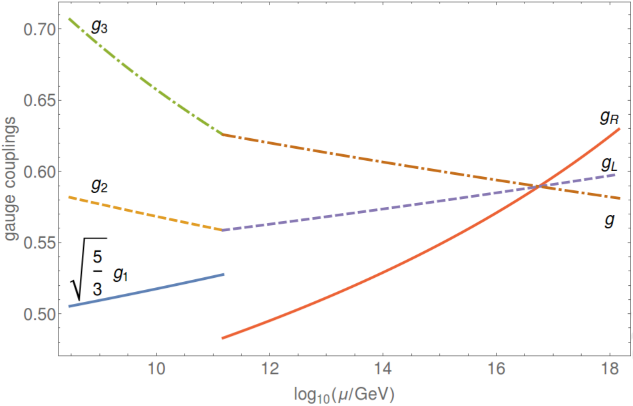
$$V(\sigma, h) = -\frac{4g_2^2}{\pi^2} f_2 \Lambda^2 (h^2 + \sigma^2) + \frac{1}{24}\lambda_h h^4 + \frac{1}{2} h^2 \sigma^2 + \frac{1}{4}\lambda_\sigma \sigma^4$$

- Instead of the notorious Higgs mass prediction from the nc Standard Model, this real scalar singlet gives a Higgs mass varying with $\rho = m_{\text{top}}/m_\nu$ and the unification scale $t = \log(\Lambda_{\text{GUT}}/M_Z)$



- This allows for $m_h = 125.5\text{GeV}$ and $m_\sigma \sim 10^{12}\text{GeV}$.

For the more general case, we have found that with $m_R \approx 1.5 \times 10^{11}$ GeV there can be unification at $\Lambda \approx 6.3 \times 10^{16}$ GeV:



Conclusion

We have arrived at a spectral Pati–Salam model that

- goes beyond the Standard Model
- has a fixed scalar sector once the finite Dirac operator has been fixed (only a few scenarios)
- exhibits grand unification for all of these scenarios
- the scalar sector has the potential to stabilize the Higgs vacuum and allow for a realistic Higgs mass.

Further phenomenological analysis: W_R, diphoton, scalar leptoquark, B-decay [Aydemir–Minic–Sun–Takeuchi 2015, 2016, 2018]

Further reading

A. Chamseddine, A. Connes, WvS.

Beyond the Spectral Standard Model: Emergence of Pati-Salam Unification. *JHEP* 11 (2013) 132. [arXiv:1304.8050]

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The spectral model of particle physics. Nieuw Archief voor de Wiskunde 5/15 (2014) 240–250.

Noncommutative Geometry and Particle Physics. Mathematical Physics Studies, Springer, 2015.

and also: `http://www.waltervansuijlekom.nl`