Applications of spectral action to particle physics.

The spectral action gives a Lagrangian $\mathcal{L}_0$ which forms the starting point for the quantum field theory.

There are quantum corrections, formally captured by path integral.

One-loop path integral: (background field method)

$$\int_{\Omega_D^1(A)} D[\hat{A}]\, e^{-Q_A(\hat{A},\hat{A})} \qquad Q_A(\hat{A},\hat{A}) = (\hat{A}, T_A(\hat{A}))$$

Write $\tilde{A} = \sum \tilde{A}_n e_n$

$$= \int \prod d\tilde{A}_n\, e^{-\sum \overline{\tilde{A}}_n (T_A)_{nm} \tilde{A}_n} = \det T_A^{-1/2}$$

One is interested in $e^{-W[A]} = \dfrac{\det T_A}{\det T_0}^{-1/2}$

$$= \left(\det T_A T_0^{-1} \right)^{-1/2}$$

Let us suppose T_A is Laplace-type, even assume $T_A = (D+A)^2$

Need regularization: "$\log \det = \operatorname{tr} \log$"

$$W[A] = -\frac{1}{2} \operatorname{tr} \left(\log (D+A)^2 - \log D^2 \right)$$

and now use $\log \frac{a}{b} = -\int_0^\infty \frac{dt}{t} \left(e^{-ta} - e^{-tb} \right)$

Zeta function <u>regularization</u>: $\frac{dt}{t} \rightsquigarrow \frac{dt}{t^{1-z}}$

$$W_z[A] = +\frac{1}{2}\tilde{\mu}^{2z}\, \mathrm{tr}\left(\int \frac{dt}{t^{1-z}} \left(e^{-t(D+A)^2} - e^{-tD^2} \right) \right)$$

$$= \frac{1}{2}\tilde{\mu}^{2z}\, \Gamma(z)\, \mathrm{tr}\left(|D+A|^{-z} - |D|^{-z} \right)$$

$$\mu^2 = e^{-\gamma_E}\tilde{\mu}^2$$

$$= \frac{1}{2}\left(\frac{1}{z} + \ln \mu^2 \right) \left(\zeta_{D+A}(z) - \zeta_D(z) \right)$$

$$= \left(\frac{1}{2z} + \ln \mu \right) \left(\zeta_{D+A}(0) - \zeta_D(0) \right) + \text{holom.}$$

$$= \left(\frac{1}{2z} + \ln \mu \right) \sum_{n \geq 0} \frac{(-1)^n}{n} f(AD^{-1})^n$$

Can be absorbed into $S_\Lambda[A]$ by replacing:

$$f(0) \mapsto f(0) - \left(\frac{1}{2z} + \ln \mu \right)$$

and this makes $f(0)$ a function of the <u>energy scale μ</u>. <u>renormalization</u>

Similar contributions from:

$$\int D[\psi]\, e^{-\langle \psi, D_A \psi \rangle} = \det D_A$$

But, if $(A_1 \oplus A_2, \mathcal{H}, D)$ we will have

$$S_\Lambda[A] \sim f(0) \cdot (\text{functional of } A_1)$$
$$+ f(0) \cdot (\text{functional of } A_2)$$

However, first $f(0) \longmapsto f(0) - c_1\left(\frac{1}{2\varepsilon} + \ln\mu\right)$

but second $f(0) \longmapsto f(0) - c_2\left(\frac{1}{2\varepsilon} + \ln\mu\right)$

$$c_1 \neq c_2 .$$

This breaks spectral description of physics models

Controlled by renormalization group equations

write $f(0)$ in terms of $g(\mu)$ couplings

$$\mu \frac{\partial}{\partial\mu} g(\mu) = \beta(g) \overset{\text{e.g.}}{=} \left(-\frac{11}{3} C_2(G) \cdots\right) \frac{g^3}{16\pi^2}$$

β_1 is different from β_2.

$\longrightarrow$ slides (sydney2019.pdf) from page 30