Spectral approach to geometry

Walter van Suijlekom

Radboud University

Spectral geometry: origins

H.A. Lorentz door Jan Veth

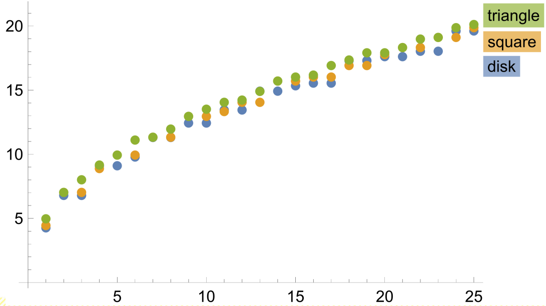
"Hierbei entseht das mathematische Problem, zu beweisen, dass die Anzahl der genügend hohen Obertöne zwischen n und $n + dn$ unabhängig von der Gestalt der Hülle und nur ihrem Volumen proportional ist."

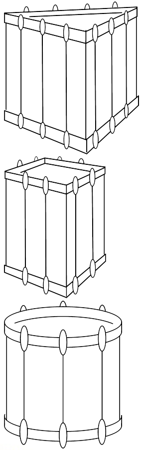
"Here arises the mathematical problem of proving that the number of sufficiently high harmonics between n and $n + dn$ is independent of the shape of the envelope and proportional only to its volume."

Weyl's Law

$$N(\Lambda) = \#\text{wave numbers } \le \Lambda$$
$$\sim \frac{\Omega_d \mathrm{Vol}(M)}{d(2\pi)^d}\Lambda^d$$

Evidence by the parabolic shapes ($\sqrt{\Lambda}$):





A spectral approach to geometry

"Can one hear the shape of a drum?" (Kac, 1966)

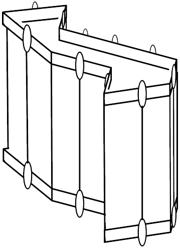
Or, more precisely, given a Riemannian manifold M, does the spectrum of wave numbers k in the Helmholtz equation

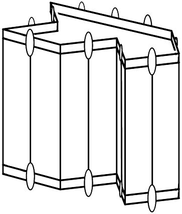
$$\boxed{\Delta_M u = k^2 u}$$

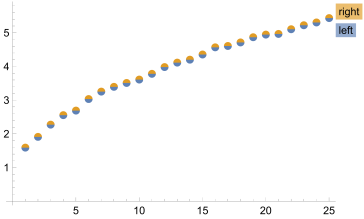
determine the geometry of M?

Similarly, for a Riemannian spin manifold and Dirac operator D_M (so that $D_M^2 = \Delta_M + \frac{1}{4}\kappa$)

Isospectral drums







so answer to Kac's question is no. More is needed...